# Package 'psd'

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**Type** Package

**Title** Adaptive, Sine-Multitaper Power Spectral Density and Cross Spectrum Estimation

**Version** 2.1.2

**Date** 2025-04-25

**Description** Produces power spectral density estimates through iterative refinement of the optimal number of sine-tapers at each frequency. This optimization procedure is based on the method of Riedel and Sidorenko (1995), which minimizes the Mean Square Error (sum of variance and bias) at each frequency, but modified for computational stability. The same procedure can now be used to calculate the cross spectrum (multivariate analyses).

**License** GPL (>= 2)

**URL** https://github.com/abarbour/psd, https://doi.org/10.1016/j.cageo.2013.09.015

**BugReports** https://github.com/abarbour/psd/issues

**Depends** R (>= 2.14.1)

**Imports** Rcpp (>= 0.11.5), RColorBrewer

**Suggests** bspec, fftw (>= 1.0.3), ggplot2 (>= 0.9), multitaper, plyr, RSEIS, rbenchmark, reshape2, knitr, rmarkdown, signal, formatR, testthat (>= 2.1.0)

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**Repository** CRAN

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# Contents

psd-package *Adaptive power spectral density estimation using optimal sine multitapers*

## Description

Estimate the power spectral density (PSD) of a timeseries using the sine multitapers, adaptively; the number of tapers (and hence the resolution and uncertainty) vary according to spectral shape. The main function to be used is `pspectrum`.

## Details

In frequency ranges where the spectrum ($S$) is relatively flat, more tapers are taken and so a higher accuracy is attained at the expense of lower frequency resolution. The program makes a pilot estimate of the spectrum, then uses Riedel and Sidorenko's (1995) estimate of the MSE (minimum square error), which is based on an estimate of the second derivative of the PSD ($S''$). The process is repeated `niter` times; further iteration may be necessary to reach convergence, or an acceptably low spectral variance. In this context the term "acceptable" is rather subjective: one can usually detect an unconverged state by a rather jagged appearance of the spectrum, but this is uncommon in our experience.

**Adaptive estimation:** The adaptive process used is as follows. A quadratic fit to the logarithm of the PSD within an adaptively determined frequency band is used to find an estimate of the local second derivative of the spectrum. This is used in an equation like R-S equation (13) for the MSE taper number, with the difference that a parabolic weighting is applied with increasing taper order. Because the FFTs of the tapered series can be found by resampling the FFT of the original time series (doubled in length and padded with zeros) only one FFT is required per series, no matter how many tapers are used. The spectra associated with the sine tapers are weighted before averaging with a parabolically varying weight. The expression for the optimal number of tapers given by R-S must be modified since it gives an unbounded result near points where $S''$ vanishes, which happens at many points in most spectra. This program restricts the rate of growth of the number of tapers so that a neighboring covering interval estimate is never completely contained in the next such interval.

**Resolution and uncertainty:** The sine multitaper adaptive process introduces a variable resolution and error in the frequency domain. See documentation for `spectral_properties` details on how these are computed.

## Author(s)

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- Jonathan Kennel (ORCID)
- Robert L. Parker

Andrew J. Barbour <andy.barbour@gmail.com>, Jonathan Kennel, and Robert L. Parker

## References

Barbour, A. J. and R. L. Parker, (2014), psd: Adaptive, sine multitaper power spectral density estimation for R, *Computers and Geosciences*, **63**, 1–8, doi: 10.1016/j.cageo.2013.09.015

Percival, D. B., and A.T. Walden (1993), Spectral analysis for physical applications, *Cambridge University Press*

Prieto, G. A., R. L. Parker, D. J. Thomson, F. L. Vernon, and R. L. Graham (2007), Reducing the bias of multitaper spectrum estimates, *Geophysical Journal International*, **171**, 1269–1281, doi: 10.1111/j.1365-246X.2007.03592.x

Riedel, K. S., & Sidorenko, A. (1995), Minimum bias multiple taper spectral estimation, *Signal Processing, IEEE Transactions on*, **43**(1), 188–195.

## See Also

Useful links:

- https://github.com/abarbour/psd
- doi:10.1016/j.cageo.2013.09.015
- Report bugs at https://github.com/abarbour/psd/issues

`pspectrum` (main function); `psdcore` and `riedsid`

---

`as.tapers` *Coerce an object into a* `'tapers'` *object.*

---

## Description

In a tapered spectrum estimation algorithm, it is necessary to enforce rules on the number of tapers that may be applied.

## Usage

```
as.tapers(
  x,
  min_taper = 1,
  max_taper = NULL,
  setspan = FALSE,
  record.last = FALSE
)

tapers(
  x,
  min_taper = 1,
  max_taper = NULL,
  setspan = FALSE,
  record.last = FALSE
)
```

## Arguments

| | |
|---|---|
| x | An object to set |
| min_taper | Set all values less than this to this. |
| max_taper | Set all values greater than this to this. |
| setspan | logical; should the tapers object be passed through minspan before being returned? |
| record.last | logical; should the x be saved to the psd-environment before coercion? |

## Details

Formal requirements enforced by this function are:

- Non-zero.
- Integer values.
- Fewer than the half-length of the spectrum.

For example, we cannot apply zero tapers (the result would be a raw periodogram) or one million tapers (that would be absurd, and violate orthogonality conditions for any series less than two million terms long!).

An object with S3 class 'tapers' is created; this will have a minimum number of tapers in each position set by min_taper, and a maximum number of tapers in each position set by max_taper. If minspan=TRUE, the bounded taper is fed through minspan which will restrict the maximum tapers to less than or equal to the half-length of the spectrum.

Various classes can be coerced into a 'tapers' object; those tested sofar include: scalar, vector, matrix, data.frame, and list.

Multiple objects are concatenated into a single vector dimension.

Enabling setspan will only override max_taper should it be larger than the half-width of the series.

## Value

An object with class 'taper'

## Note

No support (yet) for use of min_taper,max_taper as vectors, although this could be quite desirable.

## Author(s)

A.J. Barbour

## See Also

is.tapers

## Examples

```
## Not run: #REX
library(psd)

##
## Objects with class 'tapers'
##
is.tapers(as.tapers(1))
is.tapers(as.tapers(1:10))

# note dimensions
as.tapers(matrix(1:10,ncol=1))
as.tapers(list(x=1:10,y=1:30))
as.tapers( x <- data.frame(x=1:10,y=10:19) )

# change constraints
as.tapers(x, min_taper=3, max_taper=10)

# class 'character' is in-coercible; raise error
try(as.tapers(c("a","b")), silent=TRUE)


## End(Not run)#REX
```

---

`coherence` *coherence*

---

## Description

Calculate coherence from the spectra and cross-spectra. This method is the same as used in `spec.pgram`.

## Usage

```
coherence(pgram)
```

## Arguments

| | |
|---|---|
| `pgram` | `numeric array` must be multivariate |

## Value

list of coherence. For multivariate time series, a matrix containing the squared coherency between different series. Column i + (j - 1) * (j - 2)/2 contains the squared coherency between columns i and j of x, where i < j.

`ctap_loess` *Taper constraints using loess smoothing*

## Description

Taper constraints using loess smoothing

## Usage

```
ctap_loess(tapvec, ...)

## S3 method for class 'tapers'
ctap_loess(tapvec, ...)

## Default S3 method:
ctap_loess(
  tapvec,
  tapseq = NULL,
  loess.span = 0.3,
  loess.degree = 1,
  verbose = TRUE,
  ...
)
```

## Arguments

| | |
|---|---|
| `tapvec` | integer or `'tapers'` object; the number of tapers at each frequency |
| `...` | optional arguments |
| `tapseq` | numeric; positions or frequencies – necessary for smoother methods |
| `loess.span` | scalar; the span used in `loess` |
| `loess.degree` | scalar; the polynomial degree |
| `verbose` | logical; should warnings and messages be given? |

## See Also

`constrain_tapers`, `ctap_simple`

## det_vector *det_vector*

### Description

Determinant for an array

### Usage

```
det_vector(x)
```

### Arguments

| | |
|---|---|
| x | numeric array values to evaluate |

### Value

vector of determinants

## hfsnm *Noise levels found in PBO strainmeter data at seismic frequencies.*

### Description

These values represent noise levels in high frequency data ($10^{-3} - 10$ Hz) from 2009, averaged over all stations in the Anza cluster of the Plate Boundary Observatory (PBO) borehole strainmeter network, and the UCSD-style longbase laser strainmeters.

### Format

A dataframe with 141 observations on the following 4 variables:

`freq` Frequencies, in Hertz.

`P50` The 50th percentile (median) noise levels in decibels relative to $1\epsilon^2/$ Hz.

`P10` The 10th percentile noise levels also in decibels.

`meter.type` The strainmeter design type.

and 2 attributes:

`source.doi` The DOI number of the source publication.

`generator` The structure of a function which will refresh the values from the supplemental files of the original publication.

## Details

NA values in the series highlight frequency bands where the noise levels are unreliable, due to a instrumental artifact.

## Source

Barbour, A. J., and Agnew, D. C. (2011), Noise Levels on Plate Boundary Observatory Borehole Strainmeters in Southern California, *Bulletin of the Seismological Society of America*, **101**(5), 2453-2466, doi:10.1785/0120110062

## See Also

pspectrum, Tohoku, magnet

## Examples

```
data(hfsnm)
str(hfsnm)
FUN <- attr(hfsnm, "generator")
try(dat <- FUN(molten=FALSE)) # may fail without library-access to BSSA
try(all.equal(dat[,1:4], hfsnm[,1:4]))
```

---

# magnet *A single line of Project MAGNET horizontal field intensity*

---

## Description

The Project MAGNET mission provided a wealth of airborne-magnetometer data spanning the globe (Coleman, 1992). This dataset represents a single track of horizontal field intensities (a very small subset of the full collection!).

## Format

A dataframe with 2048 observations on the following 4 variables.

`km` Relative along-track distance, in kilometers. The first observation is at zero kilometers.

`raw` Raw intensities, in nanotesla.

`clean` Edited raw intensities, in nanotesla

`mdiff` The difference between `clean` and `raw` intensities, in nanotesla.

## Details

**Raw and Clean Sets:** There are non-real data points in raw MAGNET series; these are instrumental artefacts, and can severely affect power spectral density (PSD) estimates. A clean series has been included so that a comparison of PSDs may be made.

Some command like `subset(magnet, abs(mdiff) > 0)` can be used to identify the rows where edits have been made.

## Source

Project MAGNET page: https://www.ngdc.noaa.gov/geomag/proj_mag.shtml

## References

Coleman, R. J. (1992), Project Magnet high-level vector survey data reduction. In *Types and Characteristics of Data for Geomagnetic Field Modeling*, **3153**, pp. 215-248.

## See Also

pspectrum, Tohoku, hfsnm

## Examples

```
data(magnet)
summary(magnet)
```

---

# modulo_floor *Nearest value below*

---

## Description

Returns the nearest m-length value (downwards from n).

## Usage

```
modulo_floor(n, m = 2L)
```

## Arguments

| | |
|---|---|
| n | integer; the number of terms (can be a vector) |
| m | integer; the modulo term (cannot be zero) |

## Details

This function is different from nextn in that the value is floored. For example: 10 is the result for n=11,m=2 whereas nextn would give 12.

## Author(s)

A.J. Barbour

## See Also

psd-utilities; psdcore uses this to truncate series to their nearest even length (i.e., m=2).

## Examples

```
n <- 11
nextn(n) # 12
modulo_floor(n) # 10

# works on vectors too:
# defaults to m=2
modulo_floor(seq_len(n))
#[1]  0  2  2  4  4  6  6  8  8 10 10

# change the floor factor
modulo_floor(seq_len(n), 3)
#[1] 0 0 3 3 3 6 6 6 9 9 9

# zeros are not allowed for m
try(modulo_floor(n, 0))
```

---

`parabolic_weights_rcpp` *parabolic_weights_field*

---

## Description

The resampled spectrum involves summing weighted tapers; this produces the weighting factors. `parabolic_weights_rcpp` is the fastest implementation, used by `resample_fft_rcpp`, but it takes only a single value.

## Usage

```
parabolic_weights_rcpp(ntap = 1L)

parabolic_weights_field(ntap)

parabolic_weights(ntap, ...)

## S3 method for class 'tapers'
parabolic_weights(ntap, tap.index = 1L, ...)

## Default S3 method:
parabolic_weights(ntap = 1L, ...)
```

## Arguments

| | |
|---|---|
| `ntap` | integer (or `tapers` object); the number of tapers to provide weightings for. |
| `...` | optional arguments |
| `tap.index` | integer; if `ntap` is a `tapers` object, the index from which to produce a sequence of weights for |

## Details

If one has a `tapers` object, specify the `taper.index` to produce a sequence of weights up to the value at that index; the user is likely to never need to use this function though.

Weighting factors, $W$, are calculated as follows:

$$W \equiv \frac{6(n^2 - K^2)}{n(4 * n - 1)(n + 1)}$$

where $n$ is the total number of tapers, and $K$ is the integer sequence $[0, n - 1]$.

The sum of tapers should equal 1, within machine precision, when $n > 0$.

## Value

A list with the number of tapers, indices of the taper sequence, and the weights $W_N$.

## Author(s)

A.J. Barbour adapted the original algorithm (R.L. Parker), and authored the optimized versions.

## See Also

resample_fft_rcpp, psdcore, riedsid2

## Examples

```
## Not run: #REX
library(psd)
library(grDevices)
library(RColorBrewer)

##
## Show parabolic weighting factors as a function of maximum tapers
##

# maximum number of tapers
maxx <- 1e3
# sequence in logspace
xseq <- seq(from=1,to=2.8,by=0.2)

# plot palette
pal <- "Spectral"
npal <- switch(pal, RdYlBu=11, Spectral=11, Blues=9)
pal.col <- RColorBrewer::brewer.pal(npal, pal)
cols <- rev(grDevices::colorRampPalette(pal.col)(maxx))

to_df <- function(W){
  # convert parabolic results to data.frame
 with(W, data.frame(taper_seq=as.vector(taper_seq), taper_weights=as.vector(taper_weights)))
}

## a roundabout way of bootstrapping y-axis limits:
```

```
#  upper
WgtsU <- parabolic_weights(5)
DfU <- to_df(WgtsU)
#  lower
WgtsL <- parabolic_weights(maxx)
DfL <- to_df(WgtsL)

ylims <- range(pretty(dB(c(DfL$taper_weights, DfU$taper_weights)))) + c(-2,5)

# function for plotting text
TFUN <- function(Df.){
  tx <- max(Df.$taper_seq)
  ty <- mean(Df.$taper_weights)
  text(log10(tx)+0.1, dB(ty), sprintf("%i", tx), col=cols[tx])
}

# function for weighting factors and plotting
WFUN <- function(x){
  message(x)
  Wgts <- parabolic_weights(x)
  Df <- to_df(Wgts)
  lcol <- cols[x]
  lines(dB(taper_weights) ~ log10(taper_seq), Df, type="s", lwd=2, col=lcol)
  TFUN(Df)
}

## Plot parabolic weighting, in dB, colored by maximum num tapers
plot(dB(taper_weights) ~ log10(taper_seq), DfU, type="s",
     xlim=c(0, log10(maxx)+0.2),
     ylim=ylims, yaxs="i",
     col=cols[5], lwd=2,
     main="Multitaper weighting factors by maximum tapers applied",
     xlab="log10 taper sequence",
     ylab="dB")
TFUN(DfU)
invisible(lapply(round(10**xseq), FUN=WFUN))
WFUN(maxx)

##

## End(Not run)#REX
```

---

pgram_compare *Compare multitaper spectrum with cosine-tapered periodogram*

---

## Description

Plot the results of psdcore against the results of spec.pgram

## Usage

```
pgram_compare(x, ...)

## S3 method for class 'amt'
pgram_compare(
  x,
  f = NULL,
  X = NULL,
  log.freq = TRUE,
  db.spec = TRUE,
  taper = 0.2,
  ...
)
```

## Arguments

| | |
|---|---|
| x | a single psdcore object |
| ... | additional parameters (currently unused) |
| f | numeric; the frequency range to plot; optional: if not given the program will show the entire band. |
| X | object used to create x; optional: if not given the program will try and access the last copy in the environment. An attempt is made to coerce to an object of class 'ts'. |
| log.freq | logical; should frequencies be transformed with log10? Note that if f is given, the values should not already be transformed. |
| db.spec | logical; should the spectrum estimates be converted to decibels with dB? |
| taper | numeric; specifies the proportion of data to taper for the cosine periodogram. |

## Value

A list with the cosine-tapered estimates and the adaptive estimates, invisibly.

## Examples

```
set.seed(1234)
X <- rnorm(1e3)

# multitaper spectrum
p <- psdcore(X, ntaper=10)

# how does it compare to a single-cosine tapered spectrum?
pgram_compare(p)

# or in a certain band
pgram_compare(p, c(0.1,0.4))

# linear frequencies
pgram_compare(p, c(0.1,0.4), log.freq = FALSE)
```

## phase *phase*

### Description

Calculate phase from the spectra and cross spectrum. This method is the same as used in `spec.pgram`.

### Usage

```
phase(pgram)
```

### Arguments

| | |
|---|---|
| `pgram` | `numeric array` must be multivariate |

### Value

list of phase. For multivariate time series a matrix containing the cross spectrum phase between different series. The format is the same as `coherence`.

## pilot_spec *Calculate initial power spectral density estimates*

### Description

This PSD is used as the starting point – the pilot spectrum – for the adaptive estimation routine.

### Usage

```
pilot_spec(x, ...)

## S3 method for class 'ts'
pilot_spec(x, ...)

## S3 method for class 'matrix'
pilot_spec(x, x.frequency, ...)

## Default S3 method:
pilot_spec(
  x,
  x.frequency = NULL,
  ntap = NULL,
  remove.AR = NULL,
  plot = FALSE,
  verbose = FALSE,
  fast = FALSE,
  ...
)
```

## Arguments

| | |
|---|---|
| x | vector; the data series to find a pilot spectrum for |
| ... | additional parameters passed to psdcore |
| x.frequency | scalar; the sampling frequency (e.g. Hz) of the series |
| ntap | scalar; the number of tapers to apply during spectrum estimation |
| remove.AR | scalar; the max AR model to be removed from the data. |
| plot | logical; should a plot be created? |
| verbose | logical; should messages be given? |
| fast | logical; use fast method in psdcore? |

## Details

A fixed number of tapers is applied across all frequencies using psdcore, and subsequent taper-refinements are based on the spectral derivatives of this spectrum; hence, changes in the number of tapers can affect how many adaptive stages may be needed (though there are no formal convergence criteria to speak of).

The taper series of the returned spectrum is constrained using `as.tapers(..., minspan=TRUE)`.

The default behavior (`remove.AR <= 0`) is to remove the standard linear model $[f(x) = \alpha x + \beta]$ from the data; however, the user can model the effect of an autoregressive process by specifying `remove.AR`.

## Value

Invisibly, an object with class `'spec'`, and `"pilot_psd"` in the working environment.

## Removing an AR effect from the spectrum

If `remove.AR > 0` the argument is used as `AR.max` in prewhiten, from which an AR-response spectrum is calculated using the best fitting model.

If the value of `remove.AR` is too low the spectrum could become distorted, so use with care. *Note, however, that the value of* `remove.AR` *will be restricted to within the range* $[1, 100]$. If the AR order is much larger than this, it's unclear how prewhiten will perform and whether the AR model is appropriate.

*Note that this function does not produce a parametric spectrum estimation; rather, it will return the amplitude response of the best-fitting AR model as* spec.ar *would.* **Interpret these results with caution, as an AR response spectrum can be misleading.**

## Author(s)

A.J. Barbour

## See Also

psdcore, prewhiten, spec.ar

## Examples

```
## Not run: #REX
library(psd)

##
## Pilot spectrum
##

data(magnet)

## simply calculate the pilot spectrum with a few tapers
plot(pilot_spec(xc <- magnet$clean), log="dB",
     main="Pilot PSDs for MAGNET and its AR-innovations (red)")

## remove the effect of an AR model
# note: remove.AR -- the max AR model to be removed from the data
plot(pilot_spec(xc, remove.AR=10), log="dB", add=TRUE, col="red")


## End(Not run)#REX
```

---

`prewhiten` *Prepare a series for spectral estimation*

---

## Description

Remove (optionally) mean, trend, and Auto Regressive (AR) model from the original series.

## Usage

```
prewhiten(tser, ...)

## Default S3 method:
prewhiten(tser, x.fsamp = 1, x.start = c(1, 1), ...)

## S3 method for class 'ts'
prewhiten(
  tser,
  AR.max = 0L,
  detrend = TRUE,
  demean = TRUE,
  impute = TRUE,
  plot = TRUE,
  verbose = TRUE,
  ...
)
```

## Arguments

| | |
|---|---|
| tser | vector; An object to prewhiten. |
| ... | variables passed to prewhiten.ts (for non ts objects) |
| x.fsamp | sampling frequency (for non ts objects) |
| x.start | start time of observations (for non ts objects) |
| AR.max | numeric; the maximum AR order to fit. |
| detrend | logical; Should a trend (and mean) be removed? |
| demean | logical; Should a mean value be removed? |
| impute | logical; Should NA values be imputed? |
| plot | logical; Should the results be plotted? |
| verbose | logical; Should messages be printed? |

## Details

The R-S multitapers do not exhibit the remarkable spectral-leakage suppression properties of the Thomson prolate tapers, so that in spectra with large dynamic range, power bleeds from the strong peaks into neighboring frequency bands of low amplitude – spectral leakage. Prewhitening can ameliorate the problem, at least for red spectra [see Chapter 9, Percival and Walden (1993)].

The value of the AR.max argument is made absolute, after which this function has essentially two modes of operation (detailed below):

AR.max **== 0** Remove (optionally) a mean and/or linear trend.

AR.max **> 0** Remove an autoregressive model

In the second case, the time series is filtered in the time domain with a finite-impulse-response filter of AR.max terms. The filter is found by solving the Yule-Walker equations for which it is assumed the series was generated by an autoregressive process, up to order AR.max.

**Mean and trend (**AR.max == 0**):**

Power spectral density estimates can become badly biased (especially at lower frequencies) if a signal of the form $f(x) = Ax + B$ is not removed from the series. If detrend=TRUE a model of this form is removed over the entire series using a linear least-squares estimator; in this case a mean value is removed regardless of the logical state of demean. To remove *only* a mean value, set detrend=FALSE and (obviously) demean=TRUE.

**Auto Regressive (AR) innovations (**AR.max > 0**):**

When an autoregressive model is removed from a non-stationary series, the residuals are known as 'innovations', and may be stationary (or very-nearly stationary). This function fits an AR model [order at least 1, but up to and including AR(AR.max)] to the series by solving the Yule-Walker equations; however, AIC is used to estimate the highest significant order, which means that higher-order components may not necessarily be fit. The resulting innovations can be used to better estimate the stationary component of the original signal, and possibly in an interactive editing method.

Note that the method used here–solving the Yule-Walker equations–is not a true maximum likelihood estimator; hence the AIC is calculated based on the variance estimate (no determinant).

From ?ar: *In* ar.yw *the variance matrix of the innovations is computed from the fitted coefficients and the autocovariance of* x.

A quick way to determine whether this may be needed for the series is to run acf on the series, and see if significant non-zero lag correlations are found. A warning is produced if the fit returns an AR(0) fit, indicating that AR prewhitening most likely inappropriate for the series, which is apparently stationary (or very nearly so). (The innovations could end up having *higher* variance than the input series in such a case.)

*Note that* AR.max *is restricted to the range* $[1, N-1]$ *where* $N$ *is the series length.*

## Value

A list with the model fits (lm and ar objects), the linear and AR prewhitened series (ts objects), and a logical flag indicating whether the I/O has been imputed. This list includes: "lmdfit", "ardfit", "prew_lm", "prew_ar", and "imputed"

*Note that if* AR.max=0 *the AR information will exist as* NULL.

## NA values

NA values are allowed. If present, and impute=TRUE, the na.locf function in the package zoo is used twice (with and without fromLast so that lead and trailing NA values are also imputed). The function name is an acronym for "Last Observation Carried Forward", a very crude method of imputation.

## Author(s)

A.J. Barbour and Robert L. Parker

## See Also

psdcore, pspectrum

## Examples

```
## Not run: #REX
library(psd)

##
## Using prewhiten to improve spectral estimates
##

data(magnet)
mts <- ts(magnet$clean)
# add a slope
mts.slope <- mts + seq_along(mts)

# Prewhiten by removing mean+trend, and
# AR model; fit truncates the series by
# a few terms, so zero pad
mts <- prewhiten(mts.slope,  AR.max=10, zero.pad="rear")
mts.p <- mts[['prew_lm']]
```

```
mts.par <- mts[['prew_ar']]

# uniformly-tapered spectral estimates
PSD <- psdcore(mts.p, ntaper=20)
PSD.ar <- psdcore(mts.par, ntaper=20)

# remove the effect of AR model
PSD.ar[['spec']] <- PSD.ar[['spec']] / mean(PSD.ar[['spec']])
PSD[['spec']] <- PSD[['spec']] / PSD.ar[['spec']]

plot(PSD, log='dB', lwd=2, ylim=c(-5,35))
plot(PSD, log='dB', add=TRUE, lwd=2, col="red")
plot(PSD.ar, log='dB', add=TRUE, col="blue", lwd=2)


## End(Not run)#REX
```

---

psd-environment *Various environment manipulation functions.*

---

## Description

The computation of *adaptive* power spectral density estimates requires bookkeeping and non-destructive manipulation of variables. The functions here are mainly convenience wrappers designed to maintain variable separation from the .GlobalEnv environment so that no innocent variable is destroyed in the process of iteratively computing spectra. **The user should generally not be using the *setters* even though all functions exist in the namespace.**

get_psd_env_pointer is a convenience wrapper to get the environment pointer.

get_psd_env_name is a convenience wrapper to get the environment name.

psd_envRefresh will clear any variables in the environment and reset the initialization stamp.

psd_envClear clears the contents of the environment.

psd_envStatus returns a list of some information regarding the status of the environment.

psd_envList returns a listing of any assignments.

psd_envGet returns the value of variable.

psd_envAssign assigns value to variable, but does not return it.

psd_envAssignGet both assigns and returns a value.

update_adapt_history updates the adaptive estimation history list.

new_adapt_history initializes a nested-list object to store the data from each iteration.

## Usage

```
get_psd_env_pointer()

get_psd_env_name()
```

```
psd_envRefresh(verbose = TRUE)

psd_envClear()

psd_envStatus()

psd_envList()

psd_envGet(variable)

psd_envAssign(variable, value)

psd_envAssignGet(variable, value)

get_adapt_history()

last_psd()

update_adapt_history(PSD, stage, ...)

## S3 method for class 'spec'
update_adapt_history(PSD, stage, ...)

## Default S3 method:
update_adapt_history(PSD, stage, ntap = NA, freq = NULL, ...)

new_adapt_history(adapt_stages)
```

## Arguments

| | |
|---|---|
| `verbose` | logical; should messages be given? |
| `variable` | character; the name of the variable to get or assign |
| `value` | character; the name of the variable to assign |
| `PSD` | vector or object with class `'spec'`; the power spectral density estimates |
| `stage` | scalar; the current stage of the adaptive estimation procedure |
| `...` | additional arguments |
| `ntap` | vector; the tapers |
| `freq` | vector; the frequencies |
| `adapt_stages` | scalar; The number of adaptive iterations to save (excluding pilot spectrum). |

## Defaults and Initialization

One can use `get_psd_env_pointer()` and `get_psd_env_name()` to access the pointer and name of the environment, if needed.

`psd_envRefresh` should be used when a fresh environment is desired: typically only if, for example, `psdcore` is used rather than `pspectrum`.

### Assigning and Retrieving

psd_envAssign and psd_envGet perform the assignments and retrieval of objects in the environment. A convenience function, psd_envAssignGet, is included so that both assignment and retrieval may be performed at the same time. This ensures the assignment has succeeded, and the returned value is not from some other frame.

### Getters and Setters

The functions here can be classified whether the get, or set variables in the environment; some do both. Others make no modifications to the environment.

**Getter:**

- get_adapt_history
- get_psd_env_name
- get_psd_env_pointer
- psd_envGet
- psd_envList
- psd_envStatus

**Setter:**

- new_adapt_history
- psd_envAssign

**Getter and Setter:**

- psd_envAssignGet
- psd_envClear
- psd_envRefresh
- update_adapt_history

### Adaptive History

The list object for historical adapt-data may be accessed with get_adapt_history. The top names of the returned list are

stg_kopt Sequential taper vectors.

stg_psd Sequential power spectral density vectors.

freq The frequencies for each set of stg_kopt and stg_psd.

### Note

psd_envClear does *not* remove the environment–simply the assignments within it.

### See Also

psd-utilities, pspectrum

## Examples

```
## Not run: #REX
library(psd)

##
## psd working environment
##

# Get some status information about the psd working environment
psd_envStatus()

# Get a list of all variables
psd_envList()

# Pull the variable "init" into .GlobalEnv
print(x <- psd_envGet("init"))

# Pull the adaptive history into .GlobalEnv
set.seed(1234)
X <- rnorm(1e3)
pspectrum(X)
get_adapt_history()


## End(Not run)#REX
```

---

psd-normalization *Normalization of power spectral density estimates.*

---

## Description

Normalize power spectral densities from various estimators into single-sided spectra.

## Usage

```
normalize(Spec, ...)

## S3 method for class 'list'
normalize(Spec, ...)

## S3 method for class 'spec'
normalize(
  Spec,
  Fsamp = 1,
  src = c("spectrum", "double.sided", "psd", "single.sided"),
  verbose = TRUE,
  ...
)
```

```
## S3 method for class 'amt'
normalize(Spec, ...)
```

## Arguments

| | |
|---|---|
| Spec | spectrum to normalize |
| ... | (unused) additional parameters |
| Fsamp | sampling frequency |
| src | character string; the source of the spectrum estimator |
| verbose | logical; should messages be given? |

## Details

Normalizations commonly encountered for power spectra depend on it's assumed sidedness: whether the spectrum is either single- or double-sided. The normalizations performed here enforce single-sidedness, and correct as necessary.

Frequencies are assumed to be based on the Nyquist frequency (half the sampling rate). For example: If a series $X$ has sampling frequency $F_S$, then the PSD frequencies will span $[0, F_S/2]$.

For amplitudes, improper normalization can can introduce errant factors of either 1/2 or $F_S$ into the estimates, depending on the assumed sidedness. These factors can be accounted for with the `src` argument, which defaults to normalizing a double-sided spectrum.

## Value

An object with its spectral values normalized accordingly.

## Spectrum sidedness and the `src` argument

**`"double.sided"` or `"spectrum"`:**

These spectra assume frequency range of $[-F_S/2, F_S/2]$, and so are normalized by scaling by a factor of two upwards. Some estimators producing double-sided spectra:

- `stats::spectrum`
- `RSEIS::mtapspec`

**`"single.sided"` or `"psd"`:** As mentioned before, these spectra assume frequency range of $[0, F_S/2]$ and are scaled only by the inverse of the sampling rate. Some estimators producing single-sided spectra:

- `psdcore`

## Author(s)

A.J. Barbour

## See Also

`psdcore`, `spectral_properties`

## Examples

```
## Not run: #REX
library(psd)

##
## Normalization
##

# timeseries with sampling frequency **not** equal to 1:
set.seed(1234)
X <- ts(rnorm(1e3), frequency=20)

# spec.pgram: double sided
pgram <- spectrum(X)

# psdcore: single sided
PSD <- psdcore(X)

# note the normalization differences:
plot(pgram, log="dB", ylim=c(-40,10))
plot(PSD, add=TRUE, col="red", log="dB")

# A crude representation of integrated spectrum:
#   should equal variance of white noise series (~= 1)
mean(pgram[['spec']]) * max(pgram[['freq']])
mean(PSD[['spec']]) * max(PSD[['freq']])

# normalize
pgram <- normalize(pgram, src="spectrum")
PSD <- normalize(pgram, src="psd")
# replot them
plot(pgram, log="dB", ylim=c(-40,10))
plot(PSD, add=TRUE, col="red", log="dB")

# Again, integrated spectrum should be ~= 1:
mean(pgram[['spec']]) * max(pgram[['freq']])
mean(PSD[['spec']]) * max(PSD[['freq']])



## End(Not run)#REX
```

---

psd-utilities *Various utility functions.*

---

## Description

*The various utility functions are:*

na_locf is meant as a simple replacement for zoo::na.locf which carries the last observation forward; here we force both directions, meaning the first observation is carried backwards as well.

vardiff returns the variance of the first (or second) difference of the series. varddiff is a convenience wrapper to return variance for the second difference.

create_poly generates an x-y sequence compatible for use with polygon

dB returns an object converted to decibels.

vector_reshape reshapes a vector into another vector.

colvec returns the object as a vertically long vector; whereas rowvec returns the object as a horizontally long vector.

is.spec and is.amt report whether an object has class 'spec' or 'amt', as would one returned by, for example, spectrum or psdcore.

is.tapers reports whether an object has class 'tapers', as would one returned by, for example, as.tapers.

na_mat populates a matrix of specified dimensions with NA values.

zeros populate a column-wise matrix with zeros; whereas, ones populates a column-wise matrix with ones. *Note that* n *is enforced to be at least 1 for both functions.*

mod finds the modulo division of two values

## Usage

```
na_locf(x)

## S3 method for class 'matrix'
na_locf(x)

## Default S3 method:
na_locf(x)

vardiff(x, double.diff = FALSE)

varddiff(x)

## S3 method for class 'spec'
varddiff(x)

## Default S3 method:
varddiff(x)

create_poly(x, y, dy, from.lower = FALSE)

dB(Rat, invert = FALSE, pos.only = TRUE, is.power = FALSE)

vector_reshape(x, vec.shape = c("horizontal", "vertical"))

colvec(x)

rowvec(x)
```

```
is.spec(Obj)

is.amt(Obj)

is.tapers(Obj)

na_mat(nrow, ncol = 1)

zeros(nrow)

ones(nrow)

mod(x, y)
```

## Arguments

| | |
|---|---|
| x, y | objects; in create_poly these are the vectors used to create a polygon-compatible sequence (x is sorted by default); in mod these are the "numerator" and "denominator", respectively. |
| double.diff | logical; should the double difference be used instead? |
| dy | numeric; the distance from y to the top and bottom of the polygonal surfaces; see from.lower |
| from.lower | logical; should the bottom be y instead of y+dy, so that dy represents the distance from the lower surface? |
| Rat | numeric; the values – ratios – to convert to decibels (dB). |
| invert | logical; assumes Rat is already in decibels, so return ratio |
| pos.only | logical; if invert=FALSE, sets negative or zero values to NA |
| is.power | logical; should the factor of 2 be included in the decibel calculation? |
| vec.shape | choice between horizontally-long or vertically-long vector. |
| Obj | An object to test for class inheritance. |
| nrow, ncol | integer; the number of rows and/or columns to create |

## Details

Decibels are defined as $10 \log_{10} \frac{X_1}{X_2}$, unless is.power=TRUE in which $\mathrm{db}X^2 \equiv 20 \log_{10} X^2$

colvec, rowvec are simple wrapper functions to vector_reshape.

Modulo division has higher order-of-operations ranking than other arithmetic operations; hence, x + 1 %% y is equivalent to x + (1 %% y) which can produce confusing results. mod is simply a series of trunc commands which reduces the chance for unintentionally erroneous results.

## Value

vector_reshape returns a "reshaped" vector, meaning it has had it's dimensions changes so that it has either one row (if vec.shape=="horizontal"), or one column ("vertical").

is.spec, is.amt, and is.tapers return the output of inherits.

`na_mat` returns a matrix of dimensions `(nrow,ncol)` with `NA` values, the representation of which is set by `NA_real_`

`mod` returns the result of a modulo division, which is equivalent to `(x) %% (y)`.

## Note

The performance of `mod` has not been tested against the `%%` arithmetic method – it may or may not be slower for large numeric vectors.

## Author(s)

A.J. Barbour

## References

For `mod`: see Peter Dalgaard's explanation of the non-bug (#14771) I raised (instead I should've asked it on R-help): https://bugs.r-project.org/show_bug.cgi?id=14771

## See Also

`psd-package`, `as.tapers`, `modulo_floor`

## Examples

```
## Not run: #REX
library(psd)

##
## Various utilities
##

set.seed(1234)
X <- rnorm(1e2)

#
# Matrix and vector creation:
#
# NA matrix
nd <- 5
na_mat(nd)
na_mat(nd,nd-1)

# zeros
zeros(nd)

# and ones
ones(nd)

#
# Check for tapers object:
#
is.tapers(X)
```

```
is.tapers(as.tapers(X))

#
# Check for spec object:
#
PSD <- spectrum(X, plot=FALSE)
plot(PSD)
# return is class 'spec'
is.spec(PSD) # TRUE
# but the underlying structure is just a list
PSD <- unclass(PSD)
is.spec(PSD) # FALSE

#
# decibels
#
dB(1) # signal is equal <--> zero dB
sig <- 1e-10
all.equal(sig, dB(dB(sig), invert=TRUE))
pow <- sig**2
all.equal(pow, dB(dB(sig, is.power=TRUE), invert=TRUE, is.power=TRUE))

#
# Variance of difference series
#
vardiff(X) # first difference
varddiff(X) # second difference
all.equal(vardiff(X, TRUE), varddiff(X))

#
# modulo division
#
x <- 1:10
mc1a <- mod(1,2)
mc2a <- mod(1+x,2)
mc1b <- 1 %% 2
mc2b <- 1 + x %% 2
mc2c <- (1 + x) %% 2
all.equal(mc1a, mc1b) # TRUE
all.equal(mc2a, mc2b) # "Mean absolute difference: 2"
all.equal(mc2a, mc2c) # TRUE
# on a series
modulo_floor(1:10) # defaults to 2
modulo_floor(1:10, 3)


## End(Not run)#REX
```

---

psdcore *Multitaper power spectral density estimates of a series*

---

## Description

Compute power spectral density (PSD) estimates for the input series using sine multitapers. This is used by pspectrum for the adaptive estimation procedure.

## Usage

```
psdcore(X.d, ...)

## S3 method for class 'ts'
psdcore(X.d, ...)

## S3 method for class 'matrix'
psdcore(X.d, X.frq, ...)

## Default S3 method:
psdcore(
  X.d,
  X.frq = NULL,
  ntaper = as.tapers(5),
  preproc = TRUE,
  na.action = stats::na.fail,
  plot = FALSE,
  refresh = FALSE,
  verbose = FALSE,
  fast = FALSE,
  ndecimate,
  ...
)
```

## Arguments

| | |
|---|---|
| X.d | the series to estimate a spectrum for |
| ... | additional parameters |
| X.frq | scalar; the sampling information (see section Sampling) |
| ntaper | scalar, vector, or tapers; the number of sine tapers to apply at each frequency |
| preproc | logical; should X.d have a linear trend removed? |
| na.action | function to deal with NA values |
| plot | logical; should the estimates be shown compared to the spectrum-based estimates? Note that this will add some computation time, since the cosine-tapered periodogram is calculated inside pgram_compare. |
| refresh | logical; ensure a free environment prior to execution |
| verbose | logical; should warnings and messages be given? |
| fast | logical; use the faster method? |
| ndecimate | now ignored |

## Details

**Tapering:** The parameter `ntaper` specifies the number of sine tapers to be used at each frequency: equal tapers at each frequency for a scalar; otherwise, use `ntaper[j]` sine tapers at `frequency[j]`.

**Truncation:** The series, with length $N$, is necessarily truncated so that $1+N/2$ evenly spaced frequencies are returned. This truncation makes the series length "highly composite", which the discrete Fourier transform (DFT) is most efficient. The "fftw" vignette (accessed with `vignette("fftw",package="psd")`) shows how the performance of a DFT can be affected by series length.

**Decimation:** No longer supported. Setting `ndecimate` will not affect the results

**Sampling:** If `X.frq` is NULL, the value is assumed to be 1, unless `X.d` is a `'ts'` object. If `X.frq > 0` it's assumed the value represents *frequency* (e.g. Hz). If `X.frq < 0` it's assumed the value represents *interval* (e.g. seconds).

## Value

An on object of class `'amt'`,`'spec'`, which has a structure similar to a regular `'spec'` object, but with a few additional fields, invisibly.

## Author(s)

A.J. Barbour; original algorithm by R.L. Parker.

## See Also

pspectrum, riedsid, parabolic_weights, pgram_compare

## Examples

```
## Not run: #REX
library(psd)

##
## Multitaper PSD estimation
##

set.seed(1234)
X <- rnorm(1e3)

# use the defaults, and appeal to plot.spec
# sampling assumed to be 1
plot(psdcore(X))

# use more tapers, compare to stats::spectrum, and clear
# env data from the previous calculation
psdcore(X, ntaper=10, plot=TRUE, refresh=TRUE)

# change the sampling frequency to 20
psdcore(X, X.frq=20, ntaper=10, plot=TRUE, refresh=TRUE)
```

```
## End(Not run)#REX
```

---

pspectrum *Adaptive sine multitaper power spectral density estimation*

---

## Description

This is the primary function to be used in this package: it returns power spectral density estimates of a timeseries, with an optimal number of tapers at each frequency based on iterative reweighted spectral derivatives. If the object given is a multicolumn object, the cross spectrum (multivariate PSD) will be calculated using the same iterative procedure.

## Usage

```
pspectrum(x, ...)

## S3 method for class 'ts'
pspectrum(x, output_column = NULL, ...)

## S3 method for class 'matrix'
pspectrum(x, x.frqsamp, ...)

## S3 method for class 'spec'
pspectrum(x, ...)

## Default S3 method:
pspectrum(
  x,
  x.frqsamp = 1,
  ntap.init = NULL,
  niter = 3,
  output_column = NULL,
  AR = FALSE,
  Nyquist.normalize = TRUE,
  verbose = TRUE,
  no.history = FALSE,
  plot = FALSE,
  ...
)

pspectrum_basic(x, ntap.init = 7, niter = 5, verbose = TRUE, ...)

adapt_message(stage, dvar = NULL)
```

## Arguments

| | |
|---|---|
| x | vector; series to find PSD estimates for; if this is a multicolumn object, a cross spectrum will be calculated. |
| ... | Optional parameters passed to riedsid2 |
| output_column | scalar integer; If the series contains multiple columns, specify which column contains the output. The default assumes the last column is the output and the others are all inputs. |
| x.frqsamp | scalar; the sampling rate (e.g. Hz) of the series x; equivalent to frequency. |
| ntap.init | scalar; the number of sine tapers to use in the pilot spectrum estimation; if NULL then the default in pilot_spec is used. |
| niter | scalar; the number of adaptive iterations to execute after the pilot spectrum is estimated. |
| AR | logical; should the effects of an AR model be removed from the pilot spectrum? |
| Nyquist.normalize | logical; should the units be returned on Hz, rather than Nyquist? |
| verbose | logical; Should messages be given? |
| no.history | logical; Should the adaptive history *not* be saved? |
| plot | logical; Should the results be plotted? |
| stage | integer; the current adaptive stage (0 is pilot) |
| dvar | numeric; the spectral variance; see also vardiff etc |

## Details

See the **Adaptive estimation** section in the description of the psd-package for details regarding adaptive estimation.

NEW as of version 2.0: use pspectrum to calculate the cross spectrum if x is a multi-column array.

pspectrum_basic is a simplified implementation used mainly for testing.

## Value

Object with class 'spec', invisibly. It also assigns the object to "final_psd" in the working environment.

## Author(s)

A.J. Barbour adapted original by R.L. Parker

## See Also

psdcore, pilot_spec, riedsid2, prewhiten

## Examples

```
## Not run: #REX
library(psd)
library(RColorBrewer)

##
## Adaptive multitaper PSD estimation
## (see also the  "psd_overview"  vignette)
##

data(magnet)
Xr <- magnet$raw
Xc <- magnet$clean

# adaptive psd estimation (turn off diagnostic plot)
PSDr <- pspectrum(Xr, plot=FALSE)
PSDc <- pspectrum(Xc, plot=FALSE)

# plot them on the same scale
plot(PSDc, log="dB",
     main="Raw and cleaned Project MAGNET power spectral density estimates",
     lwd=3, ci.col=NA, ylim=c(0,32), yaxs="i")
plot(PSDr, log="dB", add=TRUE, lwd=3, lty=5)
text(c(0.25,0.34), c(11,24), c("Clean","Raw"), cex=1)

## Change sampling, and inspect the diagnostic plot
plot(pspectrum(Xc, niter=1, x.frqsamp=10, plot=TRUE))

## Say we forgot to assign the results: we can recover from the environment with:
PSDc_recovered <- psd_envGet("final_psd")
plot(PSDc_recovered)


## End(Not run)#REX
```

---

`rcpp_ctap_simple` *c++ implementation of the RLP constraint filter*

---

## Description

c++ implementation of the RLP constraint filter

## Usage

```
rcpp_ctap_simple(tapvec, maxslope = 1L)
```

## Arguments

| | |
|---|---|
| `tapvec` | integer or `'tapers'` object; the number of tapers at each frequency |
| `maxslope` | integer; constrain based on this maximum first difference |

`resample_fft_rcpp` *Resample an fft using varying numbers of sine tapers*

## Description

Produce an un-normalized psd based on an fft and a vector of optimal sine tapers

## Usage

```
resample_fft_rcpp(fftz, tapers, verbose = TRUE, dbl = TRUE, tapcap = 1000L)
```

## Arguments

| | |
|---|---|
| `fftz` | complex; a vector representing the dual-length `fft`; see also the `dbl` argument |
| `tapers` | integer; a vector of tapers |
| `verbose` | logical; should messages be given? |
| `dbl` | logical; should the code assume `fftz` is dual-length or single-length? |
| `tapcap` | integer; the maximum number of tapers which can be applied; note that the length is automatically limited by the length of the series. |

## Details

To produce a psd estimate with our adaptive spectrum estimation method, we need only make one fft calculation initially and then apply the weighting factors given by `parabolic_weights_rcpp`, which this function does.

## See Also

`riedsid`

## Examples

```
fftz <- complex(real=1:8, imaginary = 1:8)
taps <- 1:4
try(resample_fft_rcpp(fftz, taps))
```

`resample_mvfft` *Resample an fft using varying numbers of sine tapers*

## Description

Produce an un-normalized psd based on an fft and a vector of optimal sine tapers.

## Usage

```
resample_mvfft(fftz, tapers, verbose = TRUE, dbl = TRUE, tapcap = 10000L)
```

## Arguments

| | |
|---|---|
| `fftz` | complex; a matrix representing the dual-length `fft`; see also the `dbl` argument |
| `tapers` | integer; a vector of tapers |
| `verbose` | logical; should messages be given? |
| `dbl` | logical; should the code assume `fftz` is dual-length or single-length? |
| `tapcap` | integer; the maximum number of tapers which can be applied; note that the length is automatically limited by the length of the series. |

## Details

To produce a psd estimate with our adaptive spectrum estimation method, we need only make one fft calculation initially and then apply the weighting factors given by `parabolic_weights`, which this function does.

## Value

list that includes the auto and cross-spectral density, and the number of tapers

## See Also

`riedsid`

## Examples

```
fftz <- complex(real=1:8, imaginary = 1:8)
taps <- 1:4
try(resample_mvfft(fftz, taps))
```

`riedsid` *Constrained, optimal tapers using the Riedel & Sidorenko–Parker method*

## Description

Estimates the optimal number of tapers at each frequency of given PSD, using a modified Riedel-Sidorenko MSE recipe (RS-RLP).

## Usage

```
riedsid(PSD, ...)

## S3 method for class 'spec'
riedsid(PSD, ...)

## Default S3 method:
riedsid(
  PSD,
  ntaper = 1L,
  tapseq = NULL,
  Deriv.method = c("local_qls", "spg"),
  constrained = TRUE,
  c.method = NULL,
  verbose = TRUE,
  ...
)

riedsid2(PSD, ...)

## S3 method for class 'spec'
riedsid2(PSD, ...)

## Default S3 method:
riedsid2(
  PSD,
  ntaper = 1L,
  constrained = TRUE,
  verbose = TRUE,
  fast = FALSE,
  riedsid_column = 0L,
  ...
)
```

## Arguments

| | |
|---|---|
| `PSD` | vector or class `'amt'` or `'spec'`; the spectral values used to optimize taper numbers |

| | |
|---|---|
| ... | optional arguments passed to constrain_tapers |
| ntaper | scalar or vector; number of tapers to apply optimization |
| tapseq | vector; representing positions or frequencies (same length as PSD) |
| Deriv.method | character; choice of gradient estimation method |
| constrained | logical; apply constraints with constrain_tapers; FALSE turns off constraints |
| c.method | string; constraint method to use with constrain_tapers, only if constrained=TRUE |
| verbose | logical; should messages be printed? |
| fast | logical; use faster method? |
| riedsid_column | scalar integer; which column to use in multivariate optimization. If the value is 0 the maximum number of tapers for all columns is chosen. If the value is < 0 the minimum number of tapers for all columns is chosen. If the value is 1, 2, 3, etc. the number of tapers is based on the column selected. |

## Details

The optimization is as follows. First, weighted derivatives of the input PSD are computed. Using those derivatives the optimal number of tapers is found through the RS-RLP formulation. Constraints are then placed on the practicable number of tapers.

riedsid2 is a new (faster) implementation which does not allow for multiple constraint methods; this is the preferred function to use.

**Taper constraints:** The parameter c.method provides an option to change the method of taper constraints. A description of each may be found in the documentation for constrain_tapers.

Once can use constrained=FALSE to turn off all taper constraints; this could lead to strange behavior though.

**Spectral derivatives:** The parameter Deriv.method determines which method is used to estimate derivatives.

"local_qls" (**default**) uses quadratic weighting and local least-squares estimation; this can be slower than "spg".

"spg" uses splineGrad; then, additional arguments may be passed to control the smoothness of the derivatives (e.g spar in smooth.spline).

## Value

Object with class 'tapers'

## Warning

The "spg" can become numerically unstable, and it's not clear when it will be the preferred over the "local_qls" method, other than for efficiency's sake.

## Author(s)

A.J. Barbour adapted original by R.L. Parker

## See Also

constrain_tapers, resample_fft_rcpp, psdcore, pspectrum

## Examples

```
## Not run: #REX
library(psd)

##
## Riedel-Sidorenko-Parker taper optimization
##

set.seed(1234)
# some params
nd <- 512 # num data
ntap <- 10 # num tapers
nrm <- 40 # sharpness of the peaks rel 2*variance
#
# create a pseudo spectrum
# with broad peaks
x <- 0:(nd-1)
riex <- rnorm(nd) + nrm*abs(cos(pi*x/180) + 1.2)
riex <- riex + 8*nrm*dcauchy(x, nd/3)
riex <- riex + 5*nrm*dnorm(x, nd/2)
# some flat regions
riex[riex<25] <- 25
ried <- dB(riex, invert=TRUE)

# optimize tapers
rtap <- riedsid(riex, ntaper=ntap) # deprecation warning
rtap2 <- riedsid2(riex, ntaper=ntap)
rtap3 <- riedsid2(riex, ntaper=ntap, fast=TRUE)

# plot
op <- par(no.readonly = TRUE)
par(mfrow=c(2,1), mar=rep(1.3,4), mai=rep(0.6,4))
# ... the mock spectrum
plot(riex, type="h", xaxs="i", ylim=c(0,200),
     main='Pseudo-spectrum')

# ... tapers
plot(rtap2, col=NA, xaxs="i",
     main='Original and Optimized tapers',
     ylim=c(0,max(c(ntap, rtap,rtap2,rtap3))))
# original tapers:
abline(h=ntap, lty=2)
# optimized tapers
lines(rtap, col="red")
# 2 and 2-fast
lines(rtap2, lwd=3, col="blue")
lines(rtap3, col="cyan")
par(op)
```

```
## End(Not run)#REX
```

---

`riedsid_rcpp` *replaces time consuming portion of riedsid2*

---

## Description

replaces time consuming portion of riedsid2

## Usage

```
riedsid_rcpp(PSD, ntaper, riedsid_column = 0L)
```

## Arguments

| | |
|---|---|
| `PSD` | vector or class `'amt'` or `'spec'`; the spectral values used to optimize taper numbers |
| `ntaper` | scalar or vector; number of tapers to apply optimization |
| `riedsid_column` | scalar integer; which column to use in multivariate optimization. If the value is 0 the maximum number of tapers for all columns is chosen. If the value is < 0 the minimum number of tapers for all columns is chosen. If the value is 1, 2, 3, etc. the number of tapers is based on the column selected. |

## Value

kopt vector

---

`spec-methods` *Generic methods for objects with class 'spec'*

---

## Description

Generic methods for objects with class `'spec'`

## Usage

```
## S3 method for class 'spec'
lines(x, y = NULL, type = "l", ...)

spec_details(x, ...)

## S3 method for class 'spec'
as.data.frame(x, ...)
```

```
## S3 method for class 'spec'
as.matrix(x, ...)

## S3 method for class 'spec'
as.list(x, ...)
```

## Arguments

| | |
|---|---|
| x | a 'spec' object |
| y | optional coordinate vector for the y-axis |
| type | character; the type of plot |
| ... | optional arguments |

## Details

Objects with class 'spec' are simply lists with spectral estimates and parameters as.data.frame converts the list into a 'data.frame' with individual columns for the frequency, PSD, and taper vectors; all other information will be retained as a list in the attributes.

## Author(s)

A.J. Barbour

## Examples

```
## Not run: #REX
library(psd)

##
## Objects with class 'spec'
##

set.seed(1234)
xn <- rnorm(10)
x <- spectrum(xn, plot=FALSE)
xc <- psdcore(xn)

xdf <- as.data.frame(x)
str(xdf)
is.tapers(xdf$taper)

xdfc <- as.data.frame(xc)
str(xdfc)
is.tapers(xdfc$taper)


## End(Not run)#REX
```

`spectral_properties` *Calculate properties of multitaper power spectral density estimates*

## Description

Various spectral properties may be computed from the vector of tapers, and if necessary the sampling frequency.

## Usage

```
spectral_properties(x, ...)

## S3 method for class 'spec'
spectral_properties(x, ...)

## S3 method for class 'tapers'
spectral_properties(x, ...)

## Default S3 method:
spectral_properties(x, f.samp = 1, n.freq = NULL, p = 0.95, db.ci = FALSE, ...)
```

## Arguments

| | |
|---|---|
| `x` | object to calculate spectral properties for; or a vector of number of tapers |
| `...` | additional arguments |
| `f.samp` | numeric; the sampling frequency (e.g. Hz) of the series the tapers are for |
| `n.freq` | integer; the number of frequencies of the original spectrum (if `NULL` the length of the tapers object is assumed to be the number) |
| `p` | numeric; the coverage probability, bound within $[0, 1)$ |
| `db.ci` | logical; should the uncertainty confidence intervals be returned as decibels? |

## Details

Parameter Details:

**Uncertainty:** See `spec_confint` for details.

**Resolution:** The frequency resolution depends on the number of tapers ($K$), and is found from

$$\frac{K \cdot f_N}{N_f}$$

where $f_N$ is the Nyquist frequency and $N_f$ is the number of frequencies estimated.

**Degrees of Freedom:** There are two degrees of freedom for each taper $K$:

$$\nu = 2K$$

**Bandwidth:** The bandwidth of a multitaper estimate depends on the number of tapers. Following Walden et al (1995) the effective bandwidth is $\approx 2W$ where

$$W = \frac{K+1}{2N}$$

and $N$ is the number of terms in the series, which makes $N \cdot W$ the approximate time-bandwidth product.

## Value

A list with the following properties (and names):

- `taper`: the number of tapers
- `stderr.chi .upper, .lower, .median`: results returned from `spec_confint`
- `resolution`: effective spectral resolution
- `dof`: degrees of freedom; will be slightly inaccurate for single-taper periodograms
- `bw`: effective bandwidth of the spectrum

## Author(s)

A.J. Barbour

## See Also

`spec_confint`, `psd-package`

## Examples

```
## Not run: #REX
library(psd)

##
## Spectral properties from the number of tapers used
## (portions extracted from overview vignette)
##

#
# Theoretical uncertainties from Chi^2 distribution
#
sp <- spectral_properties(as.tapers(1:50), p=0.95, db.ci=TRUE)
par(las=1)
plot(stderr.chi.upper ~ taper, sp, type="s",
       ylim=c(-10,20), yaxs="i", xaxs="i",
       xlab=expression("number of tapers ("* nu/2 *")"), ylab="dB",
       main="Spectral uncertainties")
lines(stderr.chi.lower ~ taper, sp, type="s")
lines(stderr.chi.median ~ taper, sp, type="s", lwd=2)
lines(stderr.chi.approx ~ taper, sp, type="s", col="red",lwd=2)

#
```

```
# An example using the Project MAGNET dataset
#
data(magnet)
tapinit <- 15 # tapers
dt <- 1 # 1/km

# remove mean/trend (not really necessary but good practice; also, done internally)
ats <- prewhiten(ts(magnet$clean, deltat=dt), plot=FALSE)$prew_lm

# normal and adaptive multitaper spectra
Pspec <- psdcore(ats, dt, tapinit)
Aspec <- pspectrum(ats, dt, tapinit, niter=3, plot=FALSE)

# calculate spectral properties
spp <- spectral_properties(Pspec$taper, db.ci=TRUE)
spa <- spectral_properties(Aspec$taper, db.ci=TRUE)

# function to create polygon data, and create them
pspp <- create_poly(Pspec$freq, dB(Pspec$spec), spp$stderr.chi.approx)
psppu <- create_poly(Pspec$freq, dB(Pspec$spec), spp$stderr.chi.upper)
pspa <- create_poly(Aspec$freq, dB(Aspec$spec), spa$stderr.chi.approx)
pspau <- create_poly(Aspec$freq, dB(Aspec$spec), spa$stderr.chi.upper)

##
## Project MAGNET uncertainties
##
plot(c(0,0.5),c(-8,35),col="white",
       main="Project MAGNET Spectral Uncertainty (p > 0.95)",
       ylab="", xlab="spatial frequency, 1/km", yaxt="n", frame.plot=FALSE)
lines(c(2,1,1,2)*0.01,c(5,5,8.01,8.01)-8)
text(.05, -1.4, "3.01 dB")
polygon(psppu$xx, (psppu$yy), col="light grey", border="black", lwd=0.5)
polygon(pspp$xx, (pspp$yy), col="dark grey", border=NA)
text(0.15, 6, "With adaptive\ntaper refinement", cex=1.2)
polygon(pspau$xx, (pspau$yy)-10, col="light grey", border="black", lwd=0.5)
polygon(pspa$xx, (pspa$yy)-10, col="dark grey", border=NA)
text(0.35, 22, "Uniform tapering", cex=1.2)

##
## Project MAGNET resolution
##
frq <- Aspec$freq
relp <- dB(1/spa$resolution)
par(las=1)
plot(frq, relp,
     col="light grey",
     ylim=dB(c(1,5)),
     type="h", xaxs="i", yaxs="i",
     ylab="dB", xlab="frequency, 1/km",
     main="Project MAGNET Spectral Resolution and Uncertainty")
lines(frq, relp)
lines(frq, spp$stderr.chi.upper+relp, lwd=1.5, lty=3)
lines(frq, spa$stderr.chi.upper+relp, lwd=3, lty=2)
```

```
abline(h=dB(sqrt(vardiff(Aspec$spec))), lwd=1.5, lty=2, col="red")

##

## End(Not run)#REX
```

---

spec_confint *Confidence intervals for multitaper power spectral density estimates*

---

## Description

Confidence intervals for multitaper power spectral density estimates

## Usage

```
spec_confint(x, ...)

## S3 method for class 'spec'
spec_confint(x, ...)

## S3 method for class 'tapers'
spec_confint(x, ...)

## Default S3 method:
spec_confint(x, ...)

.spec_confint(dof, p = 0.95, as.db = FALSE, ...)
```

## Arguments

| | |
|---|---|
| x | object to calculate spectral properties |
| ... | additional arguments |
| dof | numeric; the degrees of freedom $\nu$ |
| p | numeric; the coverage probability $p$, bound within $[0, 1)$ |
| as.db | logical; should the values be returned as decibels? |

## Details

The errors are estimated from the number of degrees of freedom $\nu$ by evaluating the $\chi^2_{p,\nu}(\nu, \nu)$ distribution for an optional coverage probability $p$ (defaulting to $p = 0.95$). Additionally, the $p = 0.5$ values and an approximation from $1/\sqrt{\nu - 1}$ are returned.

A more sophisticated (and complicated) approach would be to estimate via jack-knifing (Prieto et al 2007), but this is not yet made available.

Additive uncertainties $\delta S$ are returned, such that the spectrum with confidence interval is $S \pm \delta S$.

## Value

A `data.frame` with the following properties (and names):

- `lower`: Based on upper tail probabilities ($p$)
- `upper`: Based on lower tail probabilities ($1-p$)
- `median`: Based on lower tail probabilities ($p=0.5$)
- `approx`: Approximation based on $1/\sqrt{(\nu-1)}$.

## Author(s)

A.J. Barbour; some code modified from the `spec.ci` function inside `stats::plot.spec`

## See Also

spectral_properties, psd-package, `stats::plot.spec`, dB

## Examples

```
## Not run: #REX
library(psd)

##
## Confidence intervals from taper numbers
##

sp <- spectral_properties(as.tapers(1:50), p=0.95, db.ci=TRUE)

# standard errors as a function of tapers
par(las=1)
plot(stderr.chi.upper ~ taper, sp, type="s",
       ylim=c(-10,20), yaxs="i", xaxs="i",
       xlab=expression("number of tapers ("* nu/2 *")"), ylab="dB",
       main="Spectral uncertainties")
mtext("(additive factor)", line=.3)
lines(stderr.chi.lower ~ taper, sp, type="s")
lines(stderr.chi.median ~ taper, sp, type="s", lwd=2)
lines(stderr.chi.approx ~ taper, sp, type="s", col="red",lwd=2)
# indicate K needed to reach 3 dB wide confidence interval (p=.95)
abline(v=33, lty=3)
legend("topright",
        c(expression("Based on "* chi^2 *"(p,"*nu*") and (1-p,"*nu*")"),
          expression(""* chi^2 *"(p=0.5,"*nu*")"),
          "approximation"),
       lwd=c(1,3,3),
       col=c("black","black","red"),
       bg="grey98")


## End(Not run)#REX
```

splineGrad *Numerical derivatives of a series based on its smooth-spline representation*

## Description

This computes the numerical derivatives of a spline representation of the input series; differentiation of spline curves is numerically efficient.

## Usage

```
splineGrad(dseq, dsig, ...)

## Default S3 method:
splineGrad(dseq, dsig, plot.derivs = FALSE, ...)
```

## Arguments

| | |
|---|---|
| dseq | numeric; a vector of positions for dsig. |
| dsig | numeric; a vector of values (which will have a spline fit to them). |
| ... | additional arguments passed to smooth.spline |
| plot.derivs | logical; should the derivatives be plotted? |

## Details

With smoothing, the numerical instability for "noisy" data can be drastically reduced, since spline curves are inherently (at least) twice differentiable.

## Value

A matrix with columns representing $x, f(x), f'(x), f''(x)$

## Author(s)

A.J. Barbour

## See Also

smooth.spline, constrain_tapers

## Examples

```
## Not run: #REX
library(psd)

##
## Spline gradient
##
```

```
set.seed(1234)
x <- seq(0,5*pi,by=pi/64)
y <- cos(x) #**2

splineGrad(x, y, TRUE)

# unfortunately, the presence of
# noise will affect numerical derivatives
y <- y + rnorm(length(y), sd=.1)
splineGrad(x, y, TRUE)

# so change the smoothing used in smooth.spline
splineGrad(x, y, TRUE, spar=0.2)
splineGrad(x, y, TRUE, spar=0.6)
splineGrad(x, y, TRUE, spar=1.0)


## End(Not run)#REX
```

---

`tapers-constraints` *Taper constraint methods*

---

## Description

In the Riedel-Sidorenko recipe, the number of optimal tapers at each frequency is strongly dependent on the first and second derivatives of the spectrum. It is crucial to enforce constraints on the number of actual tapers applied; this is because the derivatives of "noisy" series can be bogus.

`constrain_tapers` refines the number of tapers at each frequency.

`minspan` sets bounds on the number of tapers at each frequency.

## Usage

```
constrain_tapers(tapvec, ...)

## S3 method for class 'tapers'
constrain_tapers(tapvec, ...)

## Default S3 method:
constrain_tapers(
  tapvec,
  tapseq = NULL,
  constraint.method = c("simple.slope", "loess.smooth", "none"),
  verbose = TRUE,
  ...
)

minspan(tapvec, ...)
```

```
## S3 method for class 'tapers'
minspan(tapvec, ...)

## Default S3 method:
minspan(tapvec, Kmin = NULL, Kmax = NULL, ...)
```

## Arguments

| | |
|---|---|
| tapvec | integer or 'tapers' object; the number of tapers at each frequency |
| ... | optional arguments |
| tapseq | numeric; positions or frequencies – necessary for smoother methods |
| constraint.method | character; method to use for constraints on tapers numbers |
| verbose | logical; should warnings and messages be given? |
| Kmin | numeric; the minimum to set; default is 1 |
| Kmax | numeric; the maximum to set; default is the minimum of either (7/5 max value), or (1/2 series length) |

## Details

The method by which constrain_tapers refines tapers is set with the constraint.method argument:

- 'simple.slope' use ctap_simple
- 'loess.smooth' uses ctap_loess
- 'none' returns unbounded tapers.

minspan bounds the number of tapers to within the minimum of either the maximum number of tapers found in the object, or the half-length of the series, which is necessary because it would be nonsense to have more tapers than the length of the series.

Details of the constraint methods:

**via first differencing (the default):**

ctap_simple is the preferred constraint method. The algorithm uses first-differencing to modify the number of tapers in the previous position. Effectively, the constraint is based on a causal, 1st-order Finite Impulse-response Filter (FIR) which makes the method sensitive to rapid changes in the number of tapers; naturally, smoother spectra tend to produce less fluctuation in taper numbers, which makes this well suited for adaptive processing.

This produces, generally, the most stable results, meaning repeatedly running the constraint will not change values other than on the first execution; the same cannot be said for the other methods, which are also considerably more expensive to use.

**via LOESS smoothing:**

ctap_loess uses loess to smooth the taper vector; is can be very slow thanks to quadratic scaling.

## Value

constrain_tapers: an object with class 'tapers'; minspan: a vector

## Warning

ctap_loess results tend to be strongly dependent on the tuning parameters given to loess (for obvious reasons); hence, some effort should be given to understand their effect, and/or re-tuning them if needed.

## Author(s)

A.J. Barbour and R.L. Parker

## See Also

riedsid, ctap_simple, ctap_loess, tapers

## Examples

```
## Not run: #REX
library(psd)

##
## Taper constraint procedures
##

data(magnet)
X <- magnet$clean

##
## spectrum
PSD <- psdcore(X, ntaper=10, refresh=TRUE)
## optimize tapers
kopt <- riedsid(PSD)
kopt.loess  <- riedsid(PSD, c.method="loess.smooth")
# the preferred function:
kopt2 <- riedsid2(PSD)
#
plot(as.tapers(kopt2), ylim =c(0, 60))
lines(as.tapers(kopt.loess), col='black')
lines(as.tapers(kopt), col='black', lwd=2)

##
## To compare all the methods at once:
demo("ctap")


## End(Not run)#REX
```

tapers-methods *Generic methods for objects with class 'tapers'*

## Description

Generic methods for objects with class 'tapers'

## Usage

```
## S3 method for class 'tapers'
as.data.frame(x, ...)

data.frame.tapers(x, ...)

## S3 method for class 'tapers'
print(x, ...)

## S3 method for class 'tapers'
summary(object, ...)

## S3 method for class 'summary.tapers'
print(x, ...)

## S3 method for class 'tapers'
lines(x, lwd = 1.8, col = "red", ...)

## S3 method for class 'tapers'
points(x, pch = "_", cex = 1, ...)

## S3 method for class 'tapers'
plot(
  x,
  xi = NULL,
  color.pal = c("Blues", "Spectral"),
  ylim = NULL,
  hv.lines = FALSE,
  log.y = FALSE,
  xlab = "taper index",
  ylab = "number of tapers",
  ...
)
```

## Arguments

| | |
|---|---|
| x | tapers object |
| ... | optional arguments |

| | |
|---|---|
| object | tapers object |
| lwd | line width (default is 1.8) |
| col | color of line (default is "red") |
| pch | point character (default is "_") |
| cex | point size (default is 1) |
| xi | optional vector for indices of x |
| color.pal | color palette to use (choices are: "Blues","Spectral") |
| ylim | optional limits for y-axis |
| hv.lines | logical; should horizontal (log2) and vertical reference lines be plotted? |
| log.y | logical; should the vertical scale be logarithmic? |
| xlab, ylab | character; labels for plot axes |

## Value

plot returns a list with names: line.colors (hex values)

## Author(s)

A.J. Barbour

## See Also

as.tapers, constrain_tapers

## Examples

```
##
tap <- as.tapers(c(1:49,50:0)+rnorm(1e2))
print(tap)
print(summary(tap))
plot(tap)
# no arithmetic methods
tap <- as.tapers(tap/2)
lines(tap)
```

---

# tapers-refinement *Taper constraints using simple derivatives*

---

## Description

Taper constraints using simple derivatives

## Usage

```
ctap_simple(tapvec, ...)

## S3 method for class 'tapers'
ctap_simple(tapvec, ...)

## Default S3 method:
ctap_simple(tapvec, maxslope = 1L, ...)
```

## Arguments

| | |
|---|---|
| tapvec | integer or 'tapers' object; the number of tapers at each frequency |
| ... | optional arguments |
| maxslope | integer; constrain based on this maximum first difference |

## Author(s)

A.J. Barbour

## See Also

constrain_tapers, ctap_loess

## Examples

```
# generate some random taper series and constrain them based on slopes
set.seed(1237)
n <- 11
x <- seq_len(n)
xn <- round(runif(n,1,n))

xnf <- ctap_simple(xn, 0) # flattens out
xnc <- ctap_simple(xn, 1) # no change, already only slopes = 1
try(all.equal(xnc, xn))
xnc2 <- ctap_simple(xn, 2) # slopes = 2 only

plot(xn, type='b', pch=16, ylim=c(0,12))
grid()
abline(a=0,b=1, col='red', lty=3); abline(a=0,b=2, col='blue', lty=3)
lines(xnf, type='b', col='green')
lines(xnc, type='b', col='red')
lines(xnc2, type='b', col='blue')
lines(0.2+as.vector(psd::ctap_simple(psd::as.tapers(xn))), type='b', pch=".", col='salmon')

# more examples:
```

Tohoku *Observations of teleseismic strains from the 2011 Tohoku earthquake.*

## Description

The $M_w9$ Tohoku earthquake happened on March 11, 2011. The seismic waves were recorded at stations across the globe, including by borehole strainmeters in the Network of the Americas (NOTA), which was previously known as the Plate Boundary Observatory (PBO) network.

## Format

A dataframe with 16000 observations on the following 15 variables.

`Dts` The original datetime string, in UTC.

`areal` Areal strains

`areal.tide` Tidal correction to the areal strains.

`areal.baro` Barometric correction to the areal strains.

`gamma1` Engineering differential extensional strain: $\gamma_1$

`gamma1.tide` Tidal correction for the $\gamma_1$ strains.

`gamma1.baro` Barometric pressure correction to the $\gamma_1$ strains.

`gamma2` Engineering shear strain: $\gamma_2$.

`gamma2.tide` Tidal correction for the $\gamma_2$ strains.

`gamma2.baro` Barometric pressure correction to the $\gamma_2$ strains.

`pressure.atm` Atmospheric pressure.

`pressure.pore` Pore-fluid pressure.

`Dt` The `Dts` information converted to POSIX datetime.

`Origin.secs` The number of seconds relative to the earthquake-origin time.

`epoch` Classification based on predicted P-wave arrival: preseismic or seismic.

and 2 attributes:

`units` A list of strings regarding the units of various physical quantities given here.

`iasp` A list of source and station characteristics, including the the origin time, predicted traveltimes for P and S waves, and the geodetic information used in the traveltime calculation.

## Details

These data are for station B084, which is located approximately 8500 km away from the epicenter. Because this distance is large, the seismic waves didn't arrive at this station for more than 700 seconds after the origin time. So there is a record of pre-seismic noise included, the timeseries extends 6784 seconds prior to the origin time, and 9215 seconds after.

The data are classified with the `"epoch"` variable, which separates the series into pre-seismic and seismic data; this is defined relative to the predicted P-wave arrival time from a traveltime model.

The original dataset contained `NA` values, which were imputed using `zoo::na.locf`, which fills `NA` with the last previous observation.

## Source

High frequency strain data archive:

http://borehole.unavco.org/bsm/earthquakes/NeartheEastCoastofHonshuJapan_20110311/

## References

USGS summary page:

https://earthquake.usgs.gov/earthquakes/eventpage/official20110311054624120_30/executive

## See Also

pspectrum, hfsnm, magnet

TauP.R for an R-implementation of the traveltime calculations

## Examples

```
data(Tohoku)
str(Tohoku)
```

---

# wipp30 *Water levels from borehole WIPP30*

---

## Description

Observed water levels and barometric pressure from well WIPP30 (WIPP: Waste Isolation Pilot Plant)

## Format

A matrix with 13413 rows following 4 variables.

`time` Time (hours)

`wl` Water levels (psi)

`baro` Barometric pressure (psi)

`et` Earth tide gravity potential (nanometers/second^2)

## Details

This is the dataset used in the multivariate PSD vignette

## References

Toll, N.J., Rasmussen, T.C., (2007), Removal of Barometric Pressure Effects and Earth Tides from Observed Water Levels. *Ground Water*, **45**, 101–105, doi: 10.1111/j.1745-6584.2006.00254.x

## See Also

pspectrum

## Examples

```
data(wipp30)
summary(wipp30)
```

# Index