# Package 'multibridge'

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Title Evaluating Multinomial Order Restrictions with Bridge Sampling

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**Description** Evaluate hypotheses concerning the distribution of multinomial proportions using bridge sampling. The bridge sampling routine is able to compute Bayes factors for hypotheses that entail inequality constraints, equality constraints, free parameters, and mixtures of all three. These hypotheses are tested against the encompassing hypothesis, that all parameters vary freely or against the null hypothesis that all category proportions are equal. For more information see Sarafoglou et al. (2020) <doi:10.31234/osf.io/bux7p>.

URL https://github.com/asarafoglou/multibridge/

BugReports https://github.com/asarafoglou/multibridge/issues

License GPL-2 Encoding UTF-8 LazyData true Imports Brobdingnag, coda, mvtnorm, purrr, Rcpp (>= 0.12.17), magrittr, progress, Rdpack, stringr Suggests knitr, rmarkdown, testthat SystemRequirements GNU make, mpfr (>= 3.0.0), gmp (>= 6.2.1\_1) RoxygenNote 7.2.1 LinkingTo Rcpp VignetteBuilder knitr RdMacros Rdpack Depends R (>= 3.5.0) NeedsCompilation yes

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.adjustUpperBoundForFreeParameters Adjusts Upper Bound For Free Parameters

# Description

Corrects the upper bound for current parameter. This correction only applies for parameters that are free to vary within the restriction. Then the length of the remaining stick must be based on the largest free parameter value.

## Usage

```
.adjustUpperBoundForFreeParameters(
   theta_mat,
   k,
   upper,
   nr_mult_equal,
   smaller_values,
   larger_values,
   hyp_direction
)
```

#### Arguments

theta_mat	matrix with samples from truncated Dirichlet density	
k	current parameter index	
upper	current upper bound	
nr_mult_equal	vector of multiplicative elements of collapsed parameters	
smaller_values	index of parameters that are smaller than the current one	
larger_values	index of parameters that are larger than the current one	
hyp_direction	direction specifies whether the imposed inequality constrained imposes an increasing 'smaller') or decreasing (i.e., 'larger') trend	

## Value

adjusted upper bound

.computeLengthOfRemainingStick

Computes Length Of Remaining Stick

## Description

When applying the probit transformation on the Dirichlet samples, this function is used as part of the stick-breaking algorithm. It computes the length of the remaining stick when the current element is broken off.

## Usage

```
.computeLengthOfRemainingStick(theta_mat, k, hyp_direction)
```

## Arguments

theta_mat	matrix with samples from truncated Dirichlet density	
k	current parameter index	
hyp_direction	specifies whether the imposed inequality constrained imposes an increasing (i.e. 'smaller') or decreasing (i.e., 'larger') trend	

## Value

stick length

```
bayes_factor
```

S3 method for class 'bayes\_factor.bmult'

# Description

Extracts information about computed Bayes factors from object of class bmult

#### Usage

```
bayes_factor(x)
```

#### Arguments

х

object of class bmult as returned from mult\_bf\_informed or binom\_bf\_informed or an object of class bmult\_bridge as returned from mult\_bf\_inequality or binom\_bf\_inequality

#### Value

Returns list with three data.frames. The first dataframe bf\_table summarizes information the Bayes factor for equality and inequality constraints. The second dataframe error\_measures contains for the overall Bayes factor the approximate relative mean-squared error re2, the approximate coefficient of variation cv, and the approximate percentage error percentage. The third dataframe \$bf\_ineq\_table summarized information about the Bayes factor for inequality constraints, that is, the log marginal likelihood estimates for the constrained prior and posterior distribution. In addition, it contains for each independent Bayes factor the approximate relative mean-squared error re2

#### Note

In case the restricted hypothesis is tested against  $H_0$  four data.frames will be returned. The fourth dataframe  $bf_eq_table$  summarizes information about the Bayes factor for equality constraints compared to the encompassing hypothesis.

#### Examples

<pre>bayes_factor.bmult</pre>	Extracts information about computed Bayes factors from object of
	<i>class</i> bmult

## Description

Extracts information about computed Bayes factors from object of class bmult

#### Usage

```
## S3 method for class 'bmult'
bayes_factor(x)
```

#### Arguments

х

object of class bmult as returned from mult\_bf\_informed or binom\_bf\_informed or an object of class bmult\_bridge as returned from mult\_bf\_inequality or binom\_bf\_inequality

## Value

Returns list with three data.frames. The first dataframe bf\_table summarizes information the Bayes factor for equality and inequality constraints. The second dataframe error\_measures contains for the overall Bayes factor the approximate relative mean-squared error re2, the approximate coefficient of variation cv, and the approximate percentage error percentage. The third dataframe \$bf\_ineq\_table summarized information about the Bayes factor for inequality constraints, that is, the log marginal likelihood estimates for the constrained prior and posterior distribution. In addition, it contains for each independent Bayes factor the approximate relative mean-squared error re2

#### Note

In case the restricted hypothesis is tested against  $H_0$  four data.frames will be returned. The fourth dataframe  $bf_eq_table$  summarizes information about the Bayes factor for equality constraints compared to the encompassing hypothesis.

#### Examples

binom\_bf\_equality Computes Bayes Factors For Equality Constrained Binomial Parameters

## Description

Computes Bayes factor for equality constrained binomial parameters. Null hypothesis  $H_0$  states that binomial proportions are exactly equal or exactly equal and equal to p. Alternative hypothesis  $H_e$  states that binomial proportions are free to vary.

#### Usage

binom\_bf\_equality(x, n = NULL, a, b, p = NULL)

#### Arguments

x	a vector of counts of successes, or a two-dimensional table (or matrix) with 2 columns, giving the counts of successes and failures, respectively
n	numeric. Vector of counts of trials. Must be the same length as $x$ . Ignored if $x$ is a matrix or a table
а	numeric. Vector with alpha parameters. Must be the same length as x. Default sets all alpha parameters to 1
b	numeric. Vector with beta parameters. Must be the same length as x. Default sets all beta parameters to 1
р	numeric. Hypothesized probability of success. Must be greater than 0 and less than 1. Default sets all binomial proportions exactly equal without specifying a specific value.

## Details

The model assumes that the data in x (i.e.,  $x_1, ..., x_K$ ) are the observations of K independent binomial experiments, based on  $n_1, ..., n_K$  observations. Hence, the underlying likelihood is the product of the k = 1, ..., K individual binomial functions:

$$(x_1,...x_K) \prod Binomial(N_k,\theta_k)$$

Furthermore, the model assigns a beta distribution as prior to each model parameter (i.e., underlying binomial proportions). That is:

 $\theta_k Beta(\alpha_k, \beta_k)$ 

#### Value

Returns a data.frame containing the Bayes factors LogBFe0, BFe0, and BF0e

#### Note

The following signs can be used to encode restricted hypotheses: "<" and ">" for inequality constraints, "=" for equality constraints, "," for free parameters, and "&" for independent hypotheses. The restricted hypothesis can either be a string or a character vector. For instance, the hypothesis c("theta1 < theta2, theta3") means

- theta1 is smaller than both theta2 and theta3
- The parameters theta2 and theta3 both have theta1 as lower bound, but are not influenced by each other.

The hypothesis c("theta1 < theta2 = theta3 & theta4 > theta5") means that

Two independent hypotheses are stipulated: "theta1 < theta2 = theta3" and "theta4 > theta5"

- The restrictions on the parameters theta1, theta2, and theta3 do not influence the restrictions on the parameters theta4 and theta5.
- theta1 is smaller than theta2 and theta3
- theta2 and theta3 are assumed to be equal
- theta4 is larger than theta5

#### References

Damien P, Walker SG (2001). "Sampling truncated normal, beta, and gamma densities." *Journal of Computational and Graphical Statistics*, **10**, 206–215.

Gronau QF, Sarafoglou A, Matzke D, Ly A, Boehm U, Marsman M, Leslie DS, Forster JJ, Wagenmakers E, Steingroever H (2017). "A tutorial on bridge sampling." *Journal of Mathematical Psychology*, **81**, 80–97.

Frühwirth-Schnatter S (2004). "Estimating marginal likelihoods for mixture and Markov switching models using bridge sampling techniques." *The Econometrics Journal*, **7**, 143–167.

Sarafoglou A, Haaf JM, Ly A, Gronau QF, Wagenmakers EJ, Marsman M (2021). "Evaluating Multinomial Order Restrictions with Bridge Sampling." *Psychological Methods*.

#### See Also

Other functions to evaluate informed hypotheses: binom\_bf\_inequality(), binom\_bf\_informed(), mult\_bf\_equality(), mult\_bf\_inequality(), mult\_bf\_informed()

#### Examples

```
data(journals)
x <- journals$errors
n <- journals$nr_NHST
a <- rep(1, nrow(journals))
b <- rep(1, nrow(journals))
binom_bf_equality(x=x, n=n, a=a, b=b)</pre>
```

binom\_bf\_inequality Computes Bayes Factors For Inequality Constrained Independent Binomial Parameters

#### Description

Computes Bayes factor for inequality constrained binomial parameters using a bridge sampling routine. Restricted hypothesis  $H_r$  states that binomial proportions follow a particular trend. Alternative hypothesis  $H_e$  states that binomial proportions are free to vary. binom\_bf\_inequality

# Usage

```
binom_bf_inequality(
  samples = NULL,
  restrictions = NULL,
 x = NULL,
 n = NULL,
 Hr = NULL,
 a = rep(1, ncol(samples)),
 b = rep(1, ncol(samples)),
 factor_levels = NULL,
 prior = FALSE,
 index = 1,
 maxiter = 1000,
 seed = NULL,
 niter = 5000,
 nburnin = niter * 0.05
)
```

# Arguments

samples	matrix of dimension (nsamples x nparams) with samples from truncated Dirich- let density	
restrictions	list of class bmult_rl or of class bmult_rl_ineq as returned from generate_restriction_lis that encodes inequality constraints for each independent restriction	
x	a vector of counts of successes, or a two-dimensional table (or matrix) with 2 columns, giving the counts of successes and failures, respectively	
n	numeric. Vector of counts of trials. Must be the same length as x. Ignored if x is a matrix or a table	
Hr	string or character. Encodes the user specified informed hypothesis. Use either specified factor_levels or indices to refer to parameters. See "Note" section for details on how to formulate informed hypotheses	
а	numeric. Vector with alpha parameters. Must be the same length as x. Default sets all alpha parameters to 1	
b	numeric. Vector with beta parameters. Must be the same length as x. Default sets all beta parameters to 1	
factor_levels	character. Vector with category names. Must be the same length as x	
prior	logical. If TRUE the function will ignore the data and evaluate only the prior distribution	
index	numeric. Index of current restriction. Default is 1	
maxiter	numeric. Maximum number of iterations for the iterative updating scheme used in the bridge sampling routine. Default is 1,000 to avoid infinite loops	
seed	numeric. Sets the seed for reproducible pseudo-random number generation	
niter	numeric. Vector with number of samples to be drawn from truncated distribution	

nburnin numeric. A single value specifying the number of burn-in samples when drawing from the truncated distribution. Minimum number of burn-in samples is 10. Default is 5% of the number of samples. Burn-in samples are removed automatically after the sampling.

#### Details

The model assumes that the data in x (i.e.,  $x_1, ..., x_K$ ) are the observations of K independent binomial experiments, based on  $n_1, ..., n_K$  observations. Hence, the underlying likelihood is the product of the k = 1, ..., K individual binomial functions:

$$(x_1,...x_K) \prod Binomial(N_k,\theta_k)$$

Furthermore, the model assigns a beta distribution as prior to each model parameter (i.e., underlying binomial proportions). That is:

$$\theta_k Beta(\alpha_k, \beta_k)$$

## Value

List consisting of the following elements:

- \$eval q11: log prior or posterior evaluations for prior or posterior samples
  - q12: log proposal evaluations for prior or posterior samples
  - q21: log prior or posterior evaluations for samples from proposal
  - q22: log proposal evaluations for samples from proposal

\$niter number of iterations of the iterative updating scheme

\$logml estimate of log marginal likelihood

\$hyp evaluated inequality constrained hypothesis

- \$error\_measures re2: the approximate relative mean-squared error for the marginal likelihood estimate
  - cv: the approximate coefficient of variation for the marginal likelihood estimate (assumes that bridge estimate is unbiased)
  - percentage: the approximate percentage error of the marginal likelihood estimate

#### Note

The following signs can be used to encode restricted hypotheses: "<" and ">" for inequality constraints, "=" for equality constraints, "," for free parameters, and "&" for independent hypotheses. The restricted hypothesis can either be a string or a character vector. For instance, the hypothesis c("theta1 < theta2, theta3") means

- theta1 is smaller than both theta2 and theta3
- The parameters theta2 and theta3 both have theta1 as lower bound, but are not influenced by each other.

The hypothesis c("theta1 < theta2 = theta3 & theta4 > theta5") means that

Two independent hypotheses are stipulated: "theta1 < theta2 = theta3" and "theta4 > theta5"

#### binom\_bf\_inequality

- The restrictions on the parameters theta1, theta2, and theta3 do not influence the restrictions on the parameters theta4 and theta5.
- theta1 is smaller than theta2 and theta3
- theta2 and theta3 are assumed to be equal
- theta4 is larger than theta5

## References

Gronau QF, Sarafoglou A, Matzke D, Ly A, Boehm U, Marsman M, Leslie DS, Forster JJ, Wagenmakers E, Steingroever H (2017). "A tutorial on bridge sampling." *Journal of Mathematical Psychology*, **81**, 80–97.

Sarafoglou A, Haaf JM, Ly A, Gronau QF, Wagenmakers EJ, Marsman M (2021). "Evaluating Multinomial Order Restrictions with Bridge Sampling." *Psychological Methods*.

#### See Also

#### generate\_restriction\_list

Other functions to evaluate informed hypotheses: binom\_bf\_equality(), binom\_bf\_informed(), mult\_bf\_equality(), mult\_bf\_inequality(), mult\_bf\_informed()

#### Examples

```
# priors
a <- c(1, 1, 1, 1)
b <- c(1, 1, 1, 1)
# informed hypothesis
factor_levels <- c('theta1', 'theta2', 'theta3', 'theta4')</pre>
              <- c('theta1', '<', 'theta2', '<', 'theta3', '<', 'theta4')
Hr
results_prior <- binom_bf_inequality(Hr=Hr, a=a, b=b,</pre>
factor_levels=factor_levels, prior=TRUE, seed = 2020)
# corresponds to
cbind(exp(results_prior$logml), 1/factorial(4))
# alternative - if you have samples and a restriction list
inequalities <- generate_restriction_list(Hr=Hr, a=a,b=b,</pre>
factor_levels=factor_levels)$inequality_constraints
prior_samples <- binom_tsampling(inequalities, niter = 2e3,</pre>
prior=TRUE, seed = 2020)
results_prior <- binom_bf_inequality(prior_samples, inequalities, seed=2020)</pre>
cbind(exp(results_prior$logml), 1/factorial(4))
```

binom\_bf\_informed

## Description

Evaluates informed hypotheses on multiple binomial parameters. These hypotheses can contain (a mixture of) inequality constraints, equality constraints, and free parameters. Informed hypothesis  $H_r$  states that binomial proportions obey a particular constraint.  $H_r$  can be tested against the encompassing hypothesis  $H_e$  or the null hypothesis  $H_0$ . Encompassing hypothesis  $H_e$  states that binomial proportions are free to vary. Null hypothesis  $H_0$  states that category proportions are exactly equal.

## Usage

```
binom_bf_informed(
    x,
    n = NULL,
    Hr,
    a,
    b,
    factor_levels = NULL,
    cred_level = 0.95,
    niter = 5000,
    bf_type = "LogBFer",
    seed = NULL,
    maxiter = 1000,
    nburnin = niter * 0.05
)
```

## Arguments

x	a vector of counts of successes, or a two-dimensional table (or matrix) with 2 columns, giving the counts of successes and failures, respectively	
n	numeric. Vector of counts of trials. Must be the same length as $x$ . Ignored if $x$ is a matrix or a table	
Hr	string or character. Encodes the user specified informed hypothesis. Use either specified factor_levels or indices to refer to parameters. See "Note" section for details on how to formulate informed hypotheses	
а	numeric. Vector with alpha parameters. Must be the same length as x. Default sets all alpha parameters to 1	
b	numeric. Vector with beta parameters. Must be the same length as x. Default sets all beta parameters to 1	
factor_levels	character. Vector with category names. Must be the same length as x	
cred_level	numeric. Credible interval for the posterior point estimates. Must be a single number between 0 and 1	

niter	numeric. Vector with number of samples to be drawn from truncated distribution	
bf_type	character. The Bayes factor type. When the informed hypothesis is compared to the encompassing hypothesis, the Bayes factor type can be LogBFer, BFer, or BFre. When the informed hypothesis is compared to the null hypothesis, the Bayes factor type can be LogBFr0, BF0r, or BFr0. Default is LogBFer	
seed	numeric. Sets the seed for reproducible pseudo-random number generation	
maxiter	numeric. Maximum number of iterations for the iterative updating scheme used in the bridge sampling routine. Default is 1,000 to avoid infinite loops	
nburnin	numeric. A single value specifying the number of burn-in samples when draw- ing from the truncated distribution. Minimum number of burn-in samples is 10. Default is 5% of the number of samples. Burn-in samples are removed automat- ically after the sampling.	

#### Details

The model assumes that the data in x (i.e.,  $x_1, ..., x_K$ ) are the observations of K independent binomial experiments, based on  $n_1, ..., n_K$  observations. Hence, the underlying likelihood is the product of the k = 1, ..., K individual binomial functions:

$$(x_1,...x_K) \prod Binomial(N_k,\theta_k)$$

Furthermore, the model assigns a beta distribution as prior to each model parameter (i.e., underlying binomial proportions). That is:

$$\theta_k Beta(\alpha_k, \beta_k)$$

#### Value

List consisting of the following elements

\$bf\_list gives an overview of the Bayes factor analysis:

- bf\_type: string. Contains Bayes factor type as specified by the user
- bf: data.frame. Contains Bayes factors for all Bayes factor types
- error\_measures: data.frame. Contains for the overall Bayes factor the approximate relative mean-squared error re2, the approximate coefficient of variation cv, and the approximate percentage error percentage
- logBFe\_equalities: data.frame. Lists the log Bayes factors for all independent equality constrained hypotheses
- logBFe\_inequalities: data.frame. Lists the log Bayes factor for all independent inequality constrained hypotheses

\$cred\_level numeric. User specified credible interval

\$restrictions list that encodes informed hypothesis for each independent restriction:

- full\_model: list containing the hypothesis, parameter names, data and prior specifications for the full model.
- equality\_constraints: list containing the hypothesis, parameter names, data and prior specifications for each equality constrained hypothesis.

• inequality\_constraints: list containing the hypothesis, parameter names, data and prior specifications for each inequality constrained hypothesis. In addition, in nr\_mult\_equal and nr\_mult\_free encodes which and how many parameters are equality constraint or free, in boundaries includes the boundaries of each parameter, in nineq\_per\_hyp states the number of inequality constraint parameters per independent inequality constrained hypothesis, and in direction states the direction of the inequality constraint.

\$bridge\_output list containing output from bridge sampling function:

- eval: list containing the log prior or posterior evaluations (q11) and the log proposal evaluations (q12) for the prior or posterior samples, as well as the log prior or posterior evaluations (q21) and the log proposal evaluations (q22) for the samples from the proposal distribution
- niter: number of iterations of the iterative updating scheme
- logml: estimate of log marginal likelihood
- hyp: evaluated inequality constrained hypothesis
- error\_measures: list containing in re2 the approximate relative mean-squared error for the marginal likelihood estimate, in cv the approximate coefficient of variation for the marginal likelihood estimate (assumes that bridge estimate is unbiased), and in percentage the approximate percentage error of the marginal likelihood estimate
- \$samples list containing a list for prior samples and a list of posterior samples from truncated distributions which were used to evaluate inequality constraints. Prior and posterior samples of independent inequality constraints are again saved in separate lists. Samples are stored as matrix of dimension nsamples x nparams.

#### Note

The following signs can be used to encode restricted hypotheses: "<" and ">" for inequality constraints, "=" for equality constraints, "," for free parameters, and "&" for independent hypotheses. The restricted hypothesis can either be a string or a character vector. For instance, the hypothesis c("theta1 < theta2, theta3") means

- theta1 is smaller than both theta2 and theta3
- The parameters theta2 and theta3 both have theta1 as lower bound, but are not influenced by each other.

The hypothesis c("theta1 < theta2 = theta3 & theta4 > theta5") means that

- Two independent hypotheses are stipulated: "theta1 < theta2 = theta3" and "theta4 > theta5"
- The restrictions on the parameters theta1, theta2, and theta3 do not influence the restrictions on the parameters theta4 and theta5.
- theta1 is smaller than theta2 and theta3
- theta2 and theta3 are assumed to be equal
- theta4 is larger than theta5

#### binom\_tsampling

#### References

Damien P, Walker SG (2001). "Sampling truncated normal, beta, and gamma densities." *Journal of Computational and Graphical Statistics*, **10**, 206–215.

Gronau QF, Sarafoglou A, Matzke D, Ly A, Boehm U, Marsman M, Leslie DS, Forster JJ, Wagenmakers E, Steingroever H (2017). "A tutorial on bridge sampling." *Journal of Mathematical Psychology*, **81**, 80–97.

Frühwirth-Schnatter S (2004). "Estimating marginal likelihoods for mixture and Markov switching models using bridge sampling techniques." *The Econometrics Journal*, **7**, 143–167.

Sarafoglou A, Haaf JM, Ly A, Gronau QF, Wagenmakers EJ, Marsman M (2021). "Evaluating Multinomial Order Restrictions with Bridge Sampling." *Psychological Methods*.

## See Also

Other functions to evaluate informed hypotheses: binom\_bf\_equality(), binom\_bf\_inequality(), mult\_bf\_equality(), mult\_bf\_inequality(), mult\_bf\_informed()

## Examples

binom\_tsampling Samples From Truncated Beta Densities

#### Description

Based on specified inequality constraints, samples from truncated prior or posterior beta densities.

## Usage

```
binom_tsampling(
    inequalities,
    index = 1,
    niter = 10000,
    prior = FALSE,
    nburnin = niter * 0.05,
    seed = NULL
)
```

## Arguments

inequalities	list that contains inequality constraints for each independent inequality con- strained hypotheses. The list is created in the generate_restriction_list function	
index	numeric. If multiple independent inequality constraints are specified, this index determines for which inequality constraint samples should be drawn. Must be a single value. Default is 1	
niter	numeric. A single value specifying the number of samples. Default is set to $10,000$	
prior	logical. If TRUE ignores the data that are encoded in inequalities and thus samples from the prior distribution. Default is FALSE.	
nburnin	numeric. A single value specifying the number of burn-in samples when draw- ing from the truncated distribution. Minimum number of burn-in samples is 10. Default is 5% of the number of samples. Burn-in samples are removed automat- ically after the sampling.	
seed	numeric. Sets the seed for reproducible pseudo-random number generation	

# Details

The model assumes that the data in x (i.e.,  $x_1, ..., x_K$ ) are the observations of K independent binomial experiments, based on  $n_1, ..., n_K$  observations. Hence, the underlying likelihood is the product of the k = 1, ..., K individual binomial functions:

$$(x_1,...x_K) \prod Binomial(N_k,\theta_k)$$

Furthermore, the model assigns a beta distribution as prior to each model parameter (i.e., underlying binomial proportions). That is:

 $\theta_k Beta(\alpha_k, \beta_k)$ 

#### Value

matrix of dimension niter \* nsamples containing samples from truncated beta distributions.

## Note

When equality constraints are specified in the restricted hypothesis, this function samples from the conditional Beta distributions given that the equality constraints hold.

Only inequality constrained parameters are sampled. Free parameters or parameters that are exclusively equality constrained will be ignored.

## References

Damien P, Walker SG (2001). "Sampling truncated normal, beta, and gamma densities." *Journal of Computational and Graphical Statistics*, **10**, 206–215.

## See Also

generate\_restriction\_list

#### bridge\_output

## Examples

```
x <- c(200, 130, 40, 10)
n <- c(200, 200, 200, 200)
a <- c(1, 1, 1, 1)
b <- c(1, 1, 1, 1)
factor_levels <- c('binom1', 'binom2', 'binom3', 'binom4')
Hr <- c('binom1 > binom2 > binom3 > binom4')
# generate restriction list
inequalities <- generate_restriction_list(x=x, n=n, Hr=Hr, a=a, b=b,
factor_levels=factor_levels)$inequality_constraints
# sample from prior distribution
prior_samples <- binom_tsampling(inequalities, niter = 500,
prior=TRUE)
# sample from posterior distribution
post_samples <- binom_tsampling(inequalities, niter = 500)</pre>
```

bridge\_output

S3 method for class bridge\_output.bmult

#### Description

Extracts bridge sampling output from object of class bmult

#### Usage

bridge\_output(x)

#### Arguments

Х

object of class bmult as returned from mult\_bf\_informed or binom\_bf\_informed

#### Value

Extracts output related to the bridge sampling routine. The output contains the following elements::

**\$eval** • q11: log prior or posterior evaluations for prior or posterior samples

- q12: log proposal evaluations for prior or posterior samples
- q21: log prior or posterior evaluations for samples from proposal
- q22: log proposal evaluations for samples from proposal

\$niter number of iterations of the iterative updating scheme

\$logml estimate of log marginal likelihood

\$hyp evaluated inequality constrained hypothesis

\$error\_measures • re2: the approximate relative mean-squared error for the marginal likelihood estimate

- cv: the approximate coefficient of variation for the marginal likelihood estimate (assumes that bridge estimate is unbiased)
- percentage: the approximate percentage error of the marginal likelihood estimate

## Examples

bridge\_output.bmult Extracts bridge sampling output from object of class bmult

## Description

Extracts restriction list from an object of class bmult

## Usage

## S3 method for class 'bmult'
bridge\_output(x)

#### Arguments

```
Х
```

object of class bmult as returned from mult\_bf\_informed or binom\_bf\_informed

#### Value

Extracts restriction list and associated hypothesis from an object of class bmult

# Examples

```
# data
x <- c(3, 4, 10, 11)
n <- c(15, 12, 12, 12)
# priors
a <- c(1, 1, 1, 1)
b <- c(1, 1, 1, 1)
# informed hypothesis
factor_levels <- c('theta1', 'theta2', 'theta3', 'theta4')
Hr <- c('theta1', '<', 'theta2', '<', 'theta3', '<', 'theta4')</pre>
```

## Multinomial Case

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```
out_mult <- mult_bf_informed(x=x, Hr=Hr, a=a, factor_levels=factor_levels,
niter=1e3, seed=2020)
restriction_list <- restriction_list(out_mult)</pre>
```

generate\_restriction\_list

Creates Restriction List Based On User Specified Informed Hypothesis

## Description

Encodes the user specified informed hypothesis. It creates a separate restriction list for the full model, and all independent equality and inequality constraints. The returned list features relevant information for the transformation and sampling of the model parameters, such as information about the upper and lower bound for each parameter, and the indexes of equality constrained and free parameters.

#### Usage

generate\_restriction\_list(x = NULL, n = NULL, Hr, a, b = NULL, factor\_levels)

#### Arguments

x	a vector with data (for multinomial models) or a vector of counts of successes, or a two-dimensional table (or matrix) with 2 columns, giving the counts of successes and failures, respectively (for binomial models).
n	numeric. Vector of counts of trials. Must be the same length as $x$ . Ignored if $x$ is a matrix or a table
Hr	string or character. Encodes the user specified informed hypothesis. Use either specified factor_levels or indices to refer to parameters. See "Note" section for details on how to formulate informed hypotheses
a	numeric. Vector with concentration parameters of Dirichlet distribution (for multinomial models) or alpha parameters for independent beta distributions (for binomial models). Default sets all parameters to 1
b	numeric. Vector with beta parameters. Must be the same length as x. Default sets all beta parameters to $1$
factor_levels	character. Vector with category names. Must be the same length as x

#### Details

The restriction list can be created for both binomial and multinomial models. If multinomial models are specified, the arguments b and n should be left empty and x should not be a table or matrix.

## Value

Restriction list containing the following elements:

- \$full\_model hyp: character. Vector containing the informed hypothesis as specified by the
  user
  - parameters\_full: character. Vector containing the names for each constrained parameter
  - alpha\_full: numeric. Vector containing the concentration parameters of the Dirichlet distribution (when evaluating ordered multinomial parameters) or alpha parameters of the beta distribution (when evaluating ordered binomial parameters)
  - beta\_full: numeric. Vector containing the values of beta parameters of the beta distribution (when evaluating ordered binomial parameters)
  - counts\_full: numeric. Vector containing data values (when evaluating multinomial parameters), or number of successes (when evaluating ordered binomial parameters)
  - total\_full: numeric. Vector containing the number of observations (when evaluating ordered binomial parameters, that is, number of successes and failures)

\$equality\_constraints • hyp: list. Contains all independent equality constrained hypotheses

- parameters\_equality: character. Vector containing the names for each equality constrained parameter.
- equality\_hypotheses: list. Contains the indexes of each equality constrained parameter. Note that these indices are based on the vector of all factor levels
- alpha\_equalities: list. Contains the concentration parameters for equality constrained hypotheses (when evaluating multinomial parameters) or alpha parameters of the beta distribution (when evaluating ordered binomial parameters).
- beta\_equalities: list. Contains the values of beta parameters of the beta distribution (when evaluating ordered binomial parameters)
- counts\_equalities: list. Contains data values (when evaluating multinomial parameters), or number of successes (when evaluating ordered binomial parameters) of each equality constrained parameter
- total\_equalities1: list. Contains the number of observations of each equality constrained parameter (when evaluating ordered binomial parameters, that is, number of successes and failures)
- \$inequality\_constraints hyp: list. Contains all independent inequality constrained hypotheses
  - parameters\_inequality: list. Contains the names for each inequality constrained parameter
  - inequality\_hypotheses: list. Contains the indices of each inequality constrained parameter
  - alpha\_inequalities: list. Contains for inequality constrained hypotheses the concentration parameters of the Dirichlet distribution (when evaluating ordered multinomial parameters) or alpha parameters of the beta distribution (when evaluating ordered binomial parameters).
  - beta\_inequalities: list. Contains for inequality constrained hypotheses the values of beta parameters of the beta distribution (when evaluating ordered binomial parameters).

- counts\_inequalities: list. Contains for inequality constrained parameter data values (when evaluating multinomial parameters), or number of successes (when evaluating ordered binomial parameters).
- total\_inequalities: list. Contains for each inequality constrained parameter the number of observations (when evaluating ordered binomial parameters, that is, number of successes and failures).
- boundaries: list that lists for each inequality constrained parameter the index of parameters that serve as its upper and lower bounds. Note that these indices refer to the collapsed categories (i.e., categories after conditioning for equality constraints). If a lower or upper bound is missing, for instance because the current parameter is set to be the smallest or the largest, the bounds take the value int(0).
- nr\_mult\_equal: list. Contains multiplicative elements of collapsed categories
- nr\_mult\_free: list. Contains multiplicative elements of free parameters
- mult\_equal: list. Contains for each lower and upper bound of each inequality constrained parameter necessary multiplicative elements to recreate the implied order restriction, even for collapsed parameter values. If there is no upper or lower bound, the multiplicative element will be 0.
- nineq\_per\_hyp: numeric. Vector containing the total number of inequality constrained parameters for each independent inequality constrained hypotheses.
- direction: character. Vector containing the direction for each independent inequality constrained hypothesis. Takes the values smaller or larger.

#### Note

The following signs can be used to encode restricted hypotheses: "<" and ">" for inequality constraints, "=" for equality constraints, "," for free parameters, and "&" for independent hypotheses. The restricted hypothesis can either be a string or a character vector. For instance, the hypothesis c("theta1 < theta2, theta3") means

- theta1 is smaller than both theta2 and theta3
- The parameters theta2 and theta3 both have theta1 as lower bound, but are not influenced by each other.

The hypothesis c("theta1 < theta2 = theta3 & theta4 > theta5") means that

- Two independent hypotheses are stipulated: "theta1 < theta2 = theta3" and "theta4 > theta5"
- The restrictions on the parameters theta1, theta2, and theta3 do not influence the restrictions on the parameters theta4 and theta5.
- theta1 is smaller than theta2 and theta3
- theta2 and theta3 are assumed to be equal
- theta4 is larger than theta5

## Examples

# Restriction list for ordered multinomial
x <- c(1, 4, 1, 10)
a <- c(1, 1, 1, 1)</pre>

journals

```
factor_levels <- c('mult1', 'mult2', 'mult3', 'mult4')
Hr <- c('mult2 > mult1 , mult3 = mult4')
restrictions <- generate_restriction_list(x=x, Hr=Hr, a=a,
factor_levels=factor_levels)</pre>
```

journals

Prevalence of Statistical Reporting Errors

## Description

This data set, "journals" provides a summary of statistical reporting errors (i.e., inconsistencies between reported test statistic and reported p-value) of 16,695 research articles reporting results from null hypothesis significance testing (NHST). The selected articles were published in eight major journals in psychology between 1985 to 2013:

- Developmental Psychology (DP)
- Frontiers in Psychology (FP)
- Journal of Applied Psychology (JAP)
- Journal of Consulting and Clinical Psychology (JCCP)
- Journal of Experimental Psychology: General (JEPG)
- Journal of Personality and Social Psychology (JPSP)
- Public Library of Science (PLoS)
- Psychological Science (PS)

In total, Nuijten et al. (2016) recomputed 258,105 p-values with the R software package statcheck which extracts statistics from articles and recomputes the p-values. The anonymized dataset and the data documentation was openly available on the Open Science Framework (https://osf.io/d3ukb/; https://osf.io/c6ap2/).

# Usage

data(journals)

#### Format

A data.frame with 8 rows and 14 variables:

Variable Name	Description
journal	The journal name a research article was published in.
articles_downloaded	The number of articles downloaded per journal.
articles_with_NHST	The number of articles with NHST results.
<pre>perc_articles_with_NHST</pre>	The percentage of all downloaded articles that had NHST results.
nr_NHST	The total number of NHST results.
<pre>mean_nr_NHST_per_article_with_NHST</pre>	The mean number of NHST results per article that had at least one NHST re
<pre>mean_nr_NHST_per_article_all_included</pre>	The mean number of NHST results in all downloaded articles.
errors	The total number of errors.

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#### lifestresses

dec_errors	The total number of decision errors (i.e., an error that may have changed the
perc_errors	The percentage of all results that was an error.
perc_dec_errors	The percentage of all results that was a decision error.
<pre>perc_articles_with_errors</pre>	The percentage of all articles that had at least one error.
<pre>perc_articles_with_dec_errors</pre>	The percentage of all articles that had at least one error.
APAfactor	APA factor: number of detected NHST results / total number of detected p va

## References

Nuijten MB, Hartgerink CH, van Assen MA, Epskamp S, Wicherts JM (2016). "The prevalence of statistical reporting errors in psychology (1985–2013)." *Behavior Research Methods*, **48**, 1205–1226.

#### Examples

```
data(journals)
# Prior specification
# We assign a uniform Beta distribution on each binomial probability
a <- rep(1, 8)
b <- rep(1, 8)
x <- journals$errors
n <- journals$nr_NHST
factor_levels <- levels(journals$journal)
# restricted hypothesis
Hr1 <- c('JAP , PS , JCCP , PLOS , DP , FP , JEPG < JPSP')
out <- binom_bf_informed(x=x, n=n, Hr=Hr1, a=a, b=b,
factor_levels=factor_levels, niter = 2e3)
summary(out)</pre>
```

lifestresses Memory of Life Stresses

# Description

This data set, "lifestresses", provides the number of reported life stresses (summed across participants) that occurred in specific months prior to an interview. This data set contains the subset of 147 participants who reported one negative life event over the time span of 18 months prior to an interview. Description taken from the JASP (2020) data library.

#### Usage

data(lifestresses)

#### Format

A data.frame with 18 rows and 3 variables:

- month The month in which participants reported a stressful life event.
- stress.freq The number of participants who reported a life stress in the particular month prior to an interview.
- stress.percentage The percentage of participants who reported a life stress in the particular month prior to an interview.

#### References

Haberman SJ (1978). Analysis of qualitative data: Introductory topics, volume 1. Academic Press.

JASP Team (2022). "JASP (Version 0.16.3.0) [Computer software]." https://jasp-stats.org/.

Sarafoglou A, Haaf JM, Ly A, Gronau QF, Wagenmakers EJ, Marsman M (2021). "Evaluating Multinomial Order Restrictions with Bridge Sampling." *Psychological Methods*.

Uhlenhuth EH, Haberman SJ, Balter MD, Lipman RS (1977). "Remembering life events." In *The* origins and course of psychopathology, 117–134. Springer Verlag.

## Examples

```
data(lifestresses)
# Prior specification
# We assign a uniform Dirichlet distribution, that is, we set all
# concentration parameters to 1
              <- rep(1, 18)
а
х
              <- lifestresses$stress.freq
factor_levels <- lifestresses$month</pre>
# Test the following restricted Hypothesis:
# Hr: month1 > month2 > ... > month18
              <- paste0(1:18, collapse=">"); Hr
Hr
out <- mult_bf_informed(x=x, Hr=Hr, a=a, factor_levels=factor_levels,</pre>
niter=1e3, bf_type = 'BFre', seed = 4)
m1 <- summary(out)</pre>
m1
```

mult\_bf\_equality Computes Bayes Factors For Equality Constrained Multinomial Parameters

#### Description

Computes Bayes factor for equality constrained multinomial parameters using the standard Bayesian multinomial test. Null hypothesis  $H_0$  states that category proportions are exactly equal to those specified in p. Alternative hypothesis  $H_e$  states that category proportions are free to vary.

#### Usage

mult\_bf\_equality(x, a, p = rep(1/length(a), length(a)))

#### Arguments

х	numeric. Vector with data
a	numeric. Vector with concentration parameters of Dirichlet distribution. Must be the same length as x. Default sets all concentration parameters to 1
р	numeric. A vector of probabilities of the same length as x. Its elements must be greater than 0 and less than 1. Default is 1/K

## Details

The model assumes that data follow a multinomial distribution and assigns a Dirichlet distribution as prior for the model parameters (i.e., underlying category proportions). That is:

```
x Multinomial(N, \theta)
```

 $\theta$  Dirichlet( $\alpha$ )

#### Value

Returns a data. frame containing the Bayes factors LogBFe0, BFe0, and BF0e

#### Note

The following signs can be used to encode restricted hypotheses: "<" and ">" for inequality constraints, "=" for equality constraints, "," for free parameters, and "&" for independent hypotheses. The restricted hypothesis can either be a string or a character vector. For instance, the hypothesis c("theta1 < theta2, theta3") means

- theta1 is smaller than both theta2 and theta3
- The parameters theta2 and theta3 both have theta1 as lower bound, but are not influenced by each other.

The hypothesis c("theta1 < theta2 = theta3 & theta4 > theta5") means that

- Two independent hypotheses are stipulated: "theta1 < theta2 = theta3" and "theta4 > theta5"
- The restrictions on the parameters theta1, theta2, and theta3 do not influence the restrictions on the parameters theta4 and theta5.
- theta1 is smaller than theta2 and theta3
- theta2 and theta3 are assumed to be equal
- theta4 is larger than theta5

#### References

Damien P, Walker SG (2001). "Sampling truncated normal, beta, and gamma densities." *Journal of Computational and Graphical Statistics*, **10**, 206–215.

Gronau QF, Sarafoglou A, Matzke D, Ly A, Boehm U, Marsman M, Leslie DS, Forster JJ, Wagenmakers E, Steingroever H (2017). "A tutorial on bridge sampling." *Journal of Mathematical Psychology*, **81**, 80–97.

Frühwirth-Schnatter S (2004). "Estimating marginal likelihoods for mixture and Markov switching models using bridge sampling techniques." *The Econometrics Journal*, **7**, 143–167.

Sarafoglou A, Haaf JM, Ly A, Gronau QF, Wagenmakers EJ, Marsman M (2021). "Evaluating Multinomial Order Restrictions with Bridge Sampling." *Psychological Methods*.

#### See Also

Other functions to evaluate informed hypotheses: binom\_bf\_equality(), binom\_bf\_inequality(), binom\_bf\_informed(), mult\_bf\_inequality(), mult\_bf\_informed()

## Examples

```
data(lifestresses)
x <- lifestresses$stress.freq
a <- rep(1, nrow(lifestresses))
mult_bf_equality(x=x, a=a)</pre>
```

mult\_bf\_inequality Computes Bayes Factors For Inequality Constrained Multinomial Parameters

## Description

Computes Bayes factor for inequality constrained multinomial parameters using a bridge sampling routine. Restricted hypothesis  $H_r$  states that category proportions follow a particular trend. Alternative hypothesis  $H_e$  states that category proportions are free to vary.

#### Usage

```
mult_bf_inequality(
  samples = NULL,
  restrictions = NULL,
  x = NULL,
  Hr = NULL,
  a = rep(1, ncol(samples)),
  factor_levels = NULL,
  prior = FALSE,
  index = 1,
  maxiter = 1000,
  seed = NULL,
  niter = 5000,
  nburnin = niter * 0.05
)
```

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#### Arguments

samples	matrix of dimension nsamples x nparams with samples from truncated Dirich- let density
restrictions	list of class bmult_rl or of class bmult_rl_ineq as returned from generate_restriction_list that encodes inequality constraints for each independent restriction
х	numeric. Vector with data
Hr	string or character. Encodes the user specified informed hypothesis. Use either specified factor_levels or indices to refer to parameters. See "Note" section for details on how to formulate informed hypotheses
а	numeric. Vector with concentration parameters of Dirichlet distribution. Must be the same length as x. Default sets all concentration parameters to 1
factor_levels	character. Vector with category names. Must be the same length as x
prior	logical. If TRUE the function will ignore the data and evaluate only the prior distribution
index	numeric. Index of current restriction. Default is 1
maxiter	numeric. Maximum number of iterations for the iterative updating scheme used in the bridge sampling routine. Default is 1,000 to avoid infinite loops
seed	numeric. Sets the seed for reproducible pseudo-random number generation
niter	numeric. Vector with number of samples to be drawn from truncated distribution
nburnin	numeric. A single value specifying the number of burn-in samples when draw- ing from the truncated distribution. Minimum number of burn-in samples is 10. Default is 5% of the number of samples. Burn-in samples are removed automat- ically after the sampling.

## Details

The model assumes that data follow a multinomial distribution and assigns a Dirichlet distribution as prior for the model parameters (i.e., underlying category proportions). That is:

 $x \; Multinomial(N, \theta)$ 

 $\theta$  Dirichlet( $\alpha$ )

## Value

List consisting of the following elements:

- \$eval q11: log prior or posterior evaluations for prior or posterior samples
  - q12: log proposal evaluations for prior or posterior samples
  - q21: log prior or posterior evaluations for samples from proposal
  - q22: log proposal evaluations for samples from proposal

\$niter number of iterations of the iterative updating scheme

\$logml estimate of log marginal likelihood

\$hyp evaluated inequality constrained hypothesis

- \$error\_measures re2: the approximate relative mean-squared error for the marginal likelihood estimate
  - cv: the approximate coefficient of variation for the marginal likelihood estimate (assumes that bridge estimate is unbiased)
  - percentage: the approximate percentage error of the marginal likelihood estimate

#### Note

The following signs can be used to encode restricted hypotheses: "<" and ">" for inequality constraints, "=" for equality constraints, "," for free parameters, and "&" for independent hypotheses. The restricted hypothesis can either be a string or a character vector. For instance, the hypothesis c("theta1 < theta2, theta3") means

- theta1 is smaller than both theta2 and theta3
- The parameters theta2 and theta3 both have theta1 as lower bound, but are not influenced by each other.

The hypothesis c("theta1 < theta2 = theta3 & theta4 > theta5") means that

- Two independent hypotheses are stipulated: "theta1 < theta2 = theta3" and "theta4 > theta5"
- The restrictions on the parameters theta1, theta2, and theta3 do not influence the restrictions on the parameters theta4 and theta5.
- theta1 is smaller than theta2 and theta3
- theta2 and theta3 are assumed to be equal
- theta4 is larger than theta5

#### References

Damien P, Walker SG (2001). "Sampling truncated normal, beta, and gamma densities." *Journal of Computational and Graphical Statistics*, **10**, 206–215.

Gronau QF, Sarafoglou A, Matzke D, Ly A, Boehm U, Marsman M, Leslie DS, Forster JJ, Wagenmakers E, Steingroever H (2017). "A tutorial on bridge sampling." *Journal of Mathematical Psychology*, **81**, 80–97.

Frühwirth-Schnatter S (2004). "Estimating marginal likelihoods for mixture and Markov switching models using bridge sampling techniques." *The Econometrics Journal*, **7**, 143–167.

Sarafoglou A, Haaf JM, Ly A, Gronau QF, Wagenmakers EJ, Marsman M (2021). "Evaluating Multinomial Order Restrictions with Bridge Sampling." *Psychological Methods*.

#### See Also

#### generate\_restriction\_list

Other functions to evaluate informed hypotheses: binom\_bf\_equality(), binom\_bf\_inequality(), binom\_bf\_informed(), mult\_bf\_equality(), mult\_bf\_informed()

#### mult\_bf\_informed

## Examples

```
# priors
a <- c(1, 1, 1, 1)
# informed hypothesis
factor_levels <- c('theta1', 'theta2', 'theta3', 'theta4')</pre>
Hr
              <- c('theta1', '<', 'theta2', '<', 'theta3', '<', 'theta4')
results_prior <- mult_bf_inequality(Hr=Hr, a=a, factor_levels=factor_levels,</pre>
prior=TRUE, seed = 2020)
# corresponds to
cbind(exp(results_prior$logml), 1/factorial(4))
# alternative - if you have samples and a restriction list
inequalities <- generate_restriction_list(Hr=Hr, a=a,</pre>
factor_levels=factor_levels)$inequality_constraints
prior_samples <- mult_tsampling(inequalities, niter = 2e3,</pre>
prior=TRUE, seed = 2020)
results_prior <- mult_bf_inequality(prior_samples, inequalities, seed=2020)
cbind(exp(results_prior$logml), 1/factorial(4))
```

mult\_bf\_informed Evaluates Informed Hypotheses on Multinomial Parameters

## Description

Evaluates informed hypotheses on multinomial parameters. These hypotheses can contain (a mixture of) inequality constraints, equality constraints, and free parameters. Informed hypothesis  $H_r$ states that category proportions obey the particular constraint.  $H_r$  can be tested against the encompassing hypothesis  $H_e$  or the null hypothesis  $H_0$ . Encompassing hypothesis  $H_e$  states that category proportions are free to vary. Null hypothesis  $H_0$  states that category proportions are exactly equal.

#### Usage

```
mult_bf_informed(
    x,
    Hr,
    a = rep(1, length(x)),
    factor_levels = NULL,
    cred_level = 0.95,
    niter = 5000,
    bf_type = "LogBFer",
    seed = NULL,
    maxiter = 1000,
    nburnin = niter * 0.05
)
```

## Arguments

х	numeric. Vector with data
Hr	string or character. Encodes the user specified informed hypothesis. Use either specified factor_levels or indices to refer to parameters. See "Note" section for details on how to formulate informed hypotheses
а	numeric. Vector with concentration parameters of Dirichlet distribution. Must be the same length as x. Default sets all concentration parameters to 1
factor_levels	character. Vector with category names. Must be the same length as x
cred_level	numeric. Credible interval for the posterior point estimates. Must be a single number between 0 and 1
niter	numeric. Vector with number of samples to be drawn from truncated distribution
bf_type	character. The Bayes factor type. When the informed hypothesis is compared to the encompassing hypothesis, the Bayes factor type can be LogBFer, BFer, or BFre. When the informed hypothesis is compared to the null hypothesis, the Bayes factor type can be LogBFr0, BF0r, or BFr0. Default is LogBFer
seed	numeric. Sets the seed for reproducible pseudo-random number generation
maxiter	numeric. Maximum number of iterations for the iterative updating scheme used in the bridge sampling routine. Default is 1,000 to avoid infinite loops
nburnin	numeric. A single value specifying the number of burn-in samples when draw- ing from the truncated distribution. Minimum number of burn-in samples is 10. Default is 5% of the number of samples. Burn-in samples are removed automat- ically after the sampling.

## Details

The model assumes that data follow a multinomial distribution and assigns a Dirichlet distribution as prior for the model parameters (i.e., underlying category proportions). That is:

 $x Multinomial(N, \theta)$ 

 $\theta$  Dirichlet( $\alpha$ )

#### Value

List consisting of the following elements

\$bf\_list gives an overview of the Bayes factor analysis:

- bf\_type: string. Contains Bayes factor type as specified by the user
- bf: data.frame. Contains Bayes factors for all Bayes factor types
- error\_measures: data.frame. Contains for the overall Bayes factor the approximate relative mean-squared error re2, the approximate coefficient of variation cv, and the approximate percentage error percentage
- logBFe\_equalities: data.frame. Lists the log Bayes factors for all independent equality constrained hypotheses
- logBFe\_inequalities: data.frame. Lists the log Bayes factor for all independent inequality constrained hypotheses

\$cred\_level numeric. User specified credible interval

\$restrictions list that encodes informed hypothesis for each independent restriction:

- full\_model: list containing the hypothesis, parameter names, data and prior specifications for the full model.
- equality\_constraints: list containing the hypothesis, parameter names, data and prior specifications for each equality constrained hypothesis.
- inequality\_constraints: list containing the hypothesis, parameter names, data and prior specifications for each inequality constrained hypothesis. In addition, in nr\_mult\_equal and nr\_mult\_free encodes which and how many parameters are equality constraint or free, in boundaries includes the boundaries of each parameter, in nineq\_per\_hyp states the number of inequality constraint parameters per independent inequality constrained hypothesis, and in direction states the direction of the inequality constraint.

\$bridge\_output list containing output from bridge sampling function:

- eval: list containing the log prior or posterior evaluations (q11) and the log proposal evaluations (q12) for the prior or posterior samples, as well as the log prior or posterior evaluations (q21) and the log proposal evaluations (q22) for the samples from the proposal distribution
- niter: number of iterations of the iterative updating scheme
- logml: estimate of log marginal likelihood
- hyp: evaluated inequality constrained hypothesis
- error\_measures: list containing in re2 the approximate relative mean-squared error for the marginal likelihood estimate, in cv the approximate coefficient of variation for the marginal likelihood estimate (assumes that bridge estimate is unbiased), and in percentage the approximate percentage error of the marginal likelihood estimate
- \$samples list containing a list for prior samples and a list of posterior samples from truncated distributions which were used to evaluate inequality constraints. Prior and posterior samples of independent inequality constraints are again saved in separate lists. Samples are stored as matrix of dimension nsamples x nparams.

#### Note

The following signs can be used to encode restricted hypotheses: "<" and ">" for inequality constraints, "=" for equality constraints, "," for free parameters, and "&" for independent hypotheses. The restricted hypothesis can either be a string or a character vector. For instance, the hypothesis c("theta1 < theta2, theta3") means

- theta1 is smaller than both theta2 and theta3
- The parameters theta2 and theta3 both have theta1 as lower bound, but are not influenced by each other.

The hypothesis c("theta1 < theta2 = theta3 & theta4 > theta5") means that

- Two independent hypotheses are stipulated: "theta1 < theta2 = theta3" and "theta4 > theta5"
- The restrictions on the parameters theta1, theta2, and theta3 do not influence the restrictions on the parameters theta4 and theta5.

- theta1 is smaller than theta2 and theta3
- theta2 and theta3 are assumed to be equal
- theta4 is larger than theta5

#### References

Damien P, Walker SG (2001). "Sampling truncated normal, beta, and gamma densities." *Journal of Computational and Graphical Statistics*, **10**, 206–215.

Gronau QF, Sarafoglou A, Matzke D, Ly A, Boehm U, Marsman M, Leslie DS, Forster JJ, Wagenmakers E, Steingroever H (2017). "A tutorial on bridge sampling." *Journal of Mathematical Psychology*, **81**, 80–97.

Frühwirth-Schnatter S (2004). "Estimating marginal likelihoods for mixture and Markov switching models using bridge sampling techniques." *The Econometrics Journal*, **7**, 143–167.

Sarafoglou A, Haaf JM, Ly A, Gronau QF, Wagenmakers EJ, Marsman M (2021). "Evaluating Multinomial Order Restrictions with Bridge Sampling." *Psychological Methods*.

#### See Also

Other functions to evaluate informed hypotheses: binom\_bf\_equality(), binom\_bf\_inequality(), binom\_bf\_informed(), mult\_bf\_equality(), mult\_bf\_inequality()

#### Examples

mult\_tsampling Samples From Truncated Dirichlet Density

## Description

Based on specified inequality constraints, samples from truncated prior or posterior Dirichlet density.

## mult\_tsampling

## Usage

```
mult_tsampling(
    inequalities,
    index = 1,
    niter = 10000,
    prior = FALSE,
    nburnin = niter * 0.05,
    seed = NULL
)
```

## Arguments

inequalities	list that contains inequality constraints for each independent inequality con- strained hypotheses. The list is created in the generate_restriction_list function
index	numeric. If multiple independent inequality constraints are specified, this index determines for which inequality constraint samples should be drawn. Must be a single value. Default is 1
niter	numeric. A single value specifying the number of samples. Default is set to $10,000$
prior	logical. If TRUE ignores the data that are encoded in inequalities and thus samples from the prior distribution. Default is FALSE.
nburnin	numeric. A single value specifying the number of burn-in samples when draw- ing from the truncated distribution. Minimum number of burn-in samples is 10. Default is 5% of the number of samples. Burn-in samples are removed automat- ically after the sampling.
seed	numeric. Sets the seed for reproducible pseudo-random number generation

## Details

The model assumes that data follow a multinomial distribution and assigns a Dirichlet distribution as prior for the model parameters (i.e., underlying category proportions). That is:

 $x Multinomial(N, \theta)$ 

 $\theta \ Dirichlet(\alpha)$ 

## Value

matrix of dimension niter \* nsamples containing prior or posterior samples from truncated Dirichlet distribution.

# Note

When equality constraints are specified in the restricted hypothesis, this function samples from the conditional Dirichlet distribution given that the equality constraints hold.

Only inequality constrained parameters are sampled. Free parameters or parameters that are exclusively equality constrained will be ignored.

#### References

Damien P, Walker SG (2001). "Sampling truncated normal, beta, and gamma densities." *Journal of Computational and Graphical Statistics*, **10**, 206–215.

Sarafoglou A, Haaf JM, Ly A, Gronau QF, Wagenmakers EJ, Marsman M (2021). "Evaluating Multinomial Order Restrictions with Bridge Sampling." *Psychological Methods*.

#### See Also

generate\_restriction\_list

#### Examples

```
x <- c(200, 130, 40, 10)
a <- c(1, 1, 1, 1)
factor_levels <- c('mult1', 'mult2', 'mult3', 'mult4')
Hr <- c('mult1 > mult2 > mult3 > mult4')
# generate restriction list
inequalities <- generate_restriction_list(x=x, Hr=Hr, a=a,
factor_levels=factor_levels)$inequality_constraints
# sample from prior distribution
prior_samples <- mult_tsampling(inequalities, niter = 500, prior=TRUE)
# sample from posterior distribution
post_samples <- mult_tsampling(inequalities, niter = 500)</pre>
```

peas

#### Mendelian Laws of Inheritance

#### Description

This data set, "peas", provides the categorization of crossbreeds between a plant variety that produced round yellow peas with a plant variety that produced wrinkled green peas. This data set contains the categorization of 556 plants that were categorized either as (1) round and yellow, (2) wrinkled and yellow, (3) round and green, or (4) wrinkled and green.

#### Usage

data(peas)

### Format

A data.frame with 4 rows and 2 variables:

- peas Crossbreeds that are categorized as 'roundYellow', 'wrinkledYellow', 'roundGreen', or 'wrinkledGreen'.
- counts The number of plants assigned to a one of the crossbreed categories.

#### References

Mulder J, Wagenmakers E, Marsman M (in press). "A Generalization of the Savage-Dickey Density Ratio for Testing Equality and Order Constrained Hypotheses." *The American Statistician*.

Robertson T (1978). "Testing for and against an order restriction on multinomial parameters." *Journal of the American Statistical Association*, **73**, 197–202.

Sarafoglou A, Haaf JM, Ly A, Gronau QF, Wagenmakers EJ, Marsman M (2021). "Evaluating Multinomial Order Restrictions with Bridge Sampling." *Psychological Methods*.

## Examples

```
data("peas")
# Prior specification
# We assign a uniform Dirichlet distribution, that is, we set all
# concentration parameters to 1
a <- c(1, 1, 1, 1)
x <- peas$counts</pre>
factor_levels <- levels(peas$peas)</pre>
# Test the following mixed Hypothesis:
# Hr: roundYellow > wrinkledYellow = roundGreen > wrinkledGreen
#
# Be careful: Factor levels are usually ordered alphabetically!
# When specifying hypotheses using indexes, make sure they refer to the
# correct factor levels.
Hr <- c('1 > 2 = 3 > 4')
# To avoid mistakes, write out factor levels explicitly:
Hr <- c('roundYellow > wrinkledYellow = roundGreen > wrinkledGreen')
out <- mult_bf_informed(x=x, Hr=Hr, a=a, factor_levels=factor_levels, niter=2e3,</pre>
bf_type = 'BFre')
summary(out)
```

plot.summary.bmult *Plot estimates* 

#### Description

Plots the posterior estimates from the unconstrained multi- or binomial model.

#### Usage

```
## S3 method for class 'summary.bmult'
plot(
    x,
    main = NULL,
    xlab = NULL,
    ylab = NULL,
    xlim = NULL,
```

```
ylim = NULL,
panel.first = NULL,
....
)
```

## Arguments

x	A summary.bmult-object returned by summary().
main	character. A string used as title. Defaults to the informed hypothesis and the Bayes factor.
xlab	character. A string used as title for the x-axis.
ylab	character. A string used as title for the y-axis.
xlim, ylim	numeric vectors of length 2, giving the x and y coordinates ranges.
panel.first	expression. Evaluated after the plot axes are set up but before any plotting takes place. This can be useful for drawing background grids or null distributions.
	additional arguments passed to plot.summary.bmult-method.

## Value

Invisibly returns a data. frame with the plotted estimates.

## Examples

```
# data
x <- c(3, 4, 10, 11, 7, 30)
# priors
a <- c(1, 1, 1, 1, 1, 1)
# restricted hypothesis
factor_levels <- c('theta1', 'theta2', 'theta3', 'theta4', 'theta5',</pre>
                     'theta6')
Hr <- c('theta1', '<', 'theta2', '&', 'theta3', '=',
'theta4', ',', 'theta5', '<', 'theta6')</pre>
output_total <- mult_bf_informed(x, Hr, a, factor_levels, seed=2020,</pre>
niter=1e3, bf_type = "BFer")
plot(summary(output_total))
# data for a big Bayes factor
x <- c(3, 4, 10, 11, 7, 30) * 1000
output_total <- mult_bf_informed(x, Hr, a, factor_levels, seed=2020,</pre>
niter=1e3, bf_type = "BFre")
plot(summary(output_total))
```

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print.bmult

# Description

Prints model specification

## Usage

```
## S3 method for class 'bmult'
print(x, ...)
```

# Arguments

х	object of class bmult as returned from mult_bf_informed or binom_bf_informed
	additional arguments, currently ignored

# Value

The print methods print the model specifications and descriptives and return nothing

#### Examples

```
# data
x <- c(3, 4, 10, 11)
n <- c(15, 12, 12, 12)
# priors
a <- c(1, 1, 1, 1)
b <- c(1, 1, 1, 1)
# informed hypothesis
factor_levels <- c('theta1', 'theta2', 'theta3', 'theta4')</pre>
              <- c('theta1', '<', 'theta2', '<', 'theta3', '<', 'theta4')
Hr
## Binomial Case
out_binom <- binom_bf_informed(x=x, n=n, Hr=Hr, a=a, b=b, niter=1e3,factor_levels, seed=2020)</pre>
out_binom
## Multinomial Case
out_mult <- mult_bf_informed(x=x, Hr=Hr, a=a, niter=1e3,factor_levels, seed=2020)</pre>
out_mult
```

print.bmult\_bridge Print method for class bmult\_bridge

# Description

Prints model specification

# Usage

## S3 method for class 'bmult\_bridge'
print(x, ...)

#### Arguments

х	object of class bmult_bridge as returned from mult_bf_inequality or binom_bf_inequality
	additional arguments, currently ignored

# Value

The print methods print the results from the bridge sampling algorithm and return nothing

# Examples

print.summary.bmult print method for class summary.bmult

## Description

Prints the summary from Bayes factor analysis

### Usage

```
## S3 method for class 'summary.bmult'
print(x, ...)
```

# Arguments

Х	object of class bmult or summary.bmult as returned from summary.bmult
	additional arguments, currently ignored

# Value

The print methods print the summary from the Bayes factor analysis and returns nothing

#### Examples

```
# data
x <- c(3, 4, 10, 11)
n <- c(15, 12, 12, 12)
# priors
a <- c(1, 1, 1, 1)
b <- c(1, 1, 1, 1)
# informed hypothesis
factor_levels <- c('theta1', 'theta2', 'theta3', 'theta4')</pre>
              <- c('theta1', '<', 'theta2', '<', 'theta3', '<', 'theta4')
Hr
## Binomial Case
out_binom <- binom_bf_informed(x=x, n=n, Hr=Hr, a=a, b=b, niter=1e3,factor_levels, seed=2020)</pre>
summary(out_binom)
## Multinomial Case
out_mult <- mult_bf_informed(x=x, Hr=Hr, a=a, niter=1e3,factor_levels, seed=2020)</pre>
summary(out_mult)
```

print.summary.bmult\_bridge

print method for class summary.bmult\_bridge

## Description

Prints the summary of bridge sampling output

## Usage

```
## S3 method for class 'summary.bmult_bridge'
print(x, ...)
```

# Arguments

Х	object of class summary.bmult_bridge as returned from summary.bmult_bridge
	additional arguments, currently ignored

### Value

The print methods print the summary of the bridge sampling output

# Examples

restriction\_list S3 method for class restriction\_list.bmult

# Description

Extracts restriction list from an object of class bmult

## Usage

restriction\_list(x, ...)

#### Arguments

х	object of class bmult as returned from mult_bf_informed or binom_bf_informed
	additional arguments passed to other methods

# Value

Extracts restriction list and associated hypothesis from an object of class bmult

# Examples

```
# data
x <- c(3, 4, 10, 11)
n <- c(15, 12, 12, 12)
# priors
a <- c(1, 1, 1, 1)
b <- c(1, 1, 1, 1)
# informed hypothesis</pre>
```

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restriction\_list.bmult

Extracts restriction list from an object of class bmult

# Description

Extracts restriction list from an object of class bmult

## Usage

```
## S3 method for class 'bmult'
restriction_list(x, restrictions = "inequalities", ...)
```

#### Arguments

х	object of class bmult as returned from mult_bf_informed or binom_bf_informed
restrictions	specifies whether to extract restriction list for equalities or inequalities. Default is inequalities.
	additional arguments, currently ignored

## Value

Extracts restriction list and associated hypothesis from an object of class bmult

# Examples

samples

## Description

Extracts prior and posterior samples (if applicable) from an object of class bmult

## Usage

samples(x)

# Arguments

х

object of class bmult as returned from mult\_bf\_informed or binom\_bf\_informed

#### Value

Returns list with prior and posterior samples (if applicable) from an object of class bmult

#### Examples

samples.bmult	Extracts prior and posterior samples (if applicable) from an object of
	class bmult

# Description

Extracts prior and posterior samples (if applicable) from an object of class bmult

#### Usage

```
## S3 method for class 'bmult'
samples(x)
```

#### summary.bmult

#### Arguments

Х

object of class bmult as returned from mult\_bf\_informed or binom\_bf\_informed

# Value

Returns list with prior and posterior samples (if applicable) from an object of class bmult

# Examples

summary.bmult summary method for class bmult

## Description

Summarizes results from Bayes factor analysis

## Usage

```
## S3 method for class 'bmult'
summary(object, ...)
```

#### Arguments

object	object of class bmult as returned from mult_bf_informed or binom_bf_informed
	additional arguments, currently ignored

# Value

Invisibly returns a list which contains the Bayes factor and associated hypotheses for the full model, but also the separate for the independent equality and inequality constraints.

The summary method returns a list with the following elements:

\$hyp Vector containing the informed hypothesis as specified by the user

\$bf Contains Bayes factor

\$logmlHe Contains log marginal likelihood of the encompassing model

\$logmlH0 Contains log marginal likelihood of the null model

\$logmlHr Contains log marginal likelihood of the informed model

\$re2 Contains relative mean-square error for the Bayes factor

\$bf\_type Contains Bayes factor type as specified by the user

\$cred\_level Credible interval for the posterior point estimates.

\$prior List containing the prior parameters.

\$data List containing the data.

\$nr\_equal Number of independent equality-constrained hypotheses.

\$nr\_inequal Number of independent inequality-constrained hypotheses.

\$estimates Parameter estimates for the encompassing model

- factor\_level: Vector with category names
- alpha: Vector with posterior concentration parameters of Dirichlet distribution (for multinomial models) or alpha parameters for independent beta distributions (for binomial models)
- beta: Vector with beta parameters for independent beta distributions (for binomial models)
- lower: Lower value of credible intervals of marginal beta distributions
- median: Posterior median of marginal beta distributions
- upper: Upper value of credible intervals of marginal beta distributions

## Examples

# data

summary.bmult\_bridge summary method for class bmult\_bridge

## Description

Summarizes bridge sampling results and associated error measures

# Usage

```
## S3 method for class 'bmult_bridge'
summary(object, ...)
```

## Arguments

object	object of class bmult_bridge as returned from mult_bf_inequality or binom_bf_inequality
	additional arguments, currently ignored

# Value

Invisibly returns a list which contains the log marginal likelihood for inequality constrained category proportions and associated error terms.

# Examples

tbinom\_backtrans

## Description

Transforms samples from the real line to samples from a truncated beta density using a stickbreaking algorithm. This algorithm is suitable for mixtures of equality constrained parameters, inequality constrained parameters, and free parameters

#### Usage

```
tbinom_backtrans(xi_mat, boundaries, binom_equal, hyp_direction)
```

#### Arguments

xi_mat	matrix with samples from truncated beta density. These samples should be trans- formed, so they range over the entire real line
boundaries	list containing indices for upper and lower truncation boundaries
binom_equal	multiplicative elements for each lower and upper bound of each inequality con- strained parameter.
hyp_direction	specifies whether the imposed inequality constrained imposes an increasing (i.e., 'smaller') or decreasing (i.e., 'larger') trend

## Value

list consisting of the following elements: (1) theta\_mat: matrix with transformed samples (2) lower\_mat: matrix containing the lower bound for each parameter (3) upper\_mat: matrix containing the upper bound for each parameter

tbinom\_trans

Transforms Truncated Beta Samples To Real Line

## Description

Transforms samples from a truncated beta density to the real line using a stick-breaking algorithm. This algorithm is suitable for mixtures of equality constrained parameters, inequality constrained parameters, and free parameters

## Usage

```
tbinom_trans(theta_mat, boundaries, binom_equal, hyp_direction)
```

# tdir\_backtrans

# Arguments

theta_mat	matrix with samples from truncated beta density
boundaries	list containing indices for upper and lower truncation boundaries
binom_equal	multiplicative elements for each lower and upper bound of each inequality con- strained parameter.
hyp_direction	specifies whether the imposed inequality constrained imposes an increasing (i.e., 'smaller') or decreasing (i.e., 'larger') trend

# Value

matrix with transformed samples

tdir\_backtrans Backtransforms Samples From Real Line To Dirichlet Parameters

# Description

Transforms samples from the real line to samples from a truncated Dirichlet density using a stickbreaking algorithm. This algorithm is suitable for mixtures of equality constrained parameters, inequality constrained parameters, and free parameters

# Usage

```
tdir_backtrans(
    xi_mat,
    boundaries,
    mult_equal,
    nr_mult_equal,
    nr_mult_free,
    hyp_direction
)
```

# Arguments

xi_mat	matrix with samples from truncated Dirichlet density. These samples should be transformed, so they range over the entire real line
boundaries	list containing indices for upper and lower truncation boundaries
mult_equal	multiplicative elements for each lower and upper bound of each inequality con- strained parameter.
nr_mult_equal	vector of multiplicative elements of collapsed parameters
nr_mult_free	vector of multiplicative elements of free parameters
hyp_direction	specifies whether the imposed inequality constrained imposes an increasing (i.e., 'smaller') or decreasing (i.e., 'larger') trend

# Value

list consisting of the following elements: (1) theta\_mat: matrix with transformed samples (2) lower\_mat: matrix containing the lower bound for each parameter (3) upper\_mat: matrix containing the upper bound for each parameter

tdir\_trans

Transforms Truncated Dirichlet Samples To Real Line

# Description

Transforms samples from a truncated Dirichlet density to the real line using a stick-breaking algorithm. This algorithm is suitable for mixtures of equality constrained parameters, inequality constrained parameters, and free parameters

## Usage

```
tdir_trans(
   theta_mat,
   boundaries,
   mult_equal,
   nr_mult_equal,
   nr_mult_free,
   hyp_direction
)
```

# Arguments

theta_mat	matrix with samples from truncated Dirichlet density
boundaries	list containing indices for upper and lower truncation boundaries
mult_equal	multiplicative elements for each lower and upper bound of each inequality con- strained parameter.
nr_mult_equal	vector of multiplicative elements of collapsed parameters
nr_mult_free	vector of multiplicative elements of free parameters
hyp_direction	specifies whether the imposed inequality constrained imposes an increasing (i.e., 'smaller') or decreasing (i.e., 'larger') trend

# Value

matrix with transformed samples

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