

# Package ‘mniw’

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**Type** Package

**Title** The Matrix-Normal Inverse-Wishart Distribution

**Version** 1.0.2

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**Description** Density evaluation and random number generation for the Matrix-Normal Inverse-Wishart (MNIW) distribution, as well as the the Matrix-Normal, Matrix-T, Wishart, and Inverse-Wishart distributions. Core calculations are implemented in a portable (header-only) C++ library, with matrix manipulations using the 'Eigen' library for linear algebra. Also provided is a Gibbs sampler for Bayesian inference on a random-effects model with multivariate normal observations.

**URL** <https://github.com/mlysy/mniw/>, <https://mlysy.github.io/mniw/>

**BugReports** <https://github.com/mlysy/mniw/issues>

**License** GPL-3

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## Contents

mniw-package . . . . .	2
crossprodV . . . . .	3
Hospitals . . . . .	3
MatrixNormal-dist . . . . .	4
MatrixT-dist . . . . .	5
MNIW-dist . . . . .	6
MultiNormal-dist . . . . .	7
rRxNorm . . . . .	8
RxNormLM . . . . .	9
Wishart-dist . . . . .	13

## Index

16

mniw-package

*Tools for the Matrix-Normal Inverse-Wishart distribution.*

## Description

Density evaluation and random number generation for the Matrix-Normal Inverse-Wishart (MNIW) distribution, as well as its constituent distributions, i.e., the Matrix-Normal, Matrix-T, Wishart, and Inverse-Wishart distributions.

## Details

The Matrix-Normal Inverse-Wishart (MNIW) distribution  $(\mathbf{X}, \mathbf{V}) \sim \text{MNIW}(\boldsymbol{\Lambda}, \boldsymbol{\Sigma}, \boldsymbol{\Psi}, \nu)$  on random matrices  $\mathbf{X}_{p \times q}$  and symmetric positive-definite  $\mathbf{V}_{q \times q}$  is defined as

$$\begin{aligned} \mathbf{V} &\sim \text{Inverse-Wishart}(\boldsymbol{\Psi}, \nu) \\ \mathbf{X} | \mathbf{V} &\sim \text{Matrix-Normal}(\boldsymbol{\Lambda}, \boldsymbol{\Sigma}, \mathbf{V}), \end{aligned}$$

where the Matrix-Normal distribution is defined as the multivariate normal

$$\text{vec}(\mathbf{X}) \sim \mathcal{N}(\text{vec}(\boldsymbol{\Lambda}), \mathbf{V} \otimes \boldsymbol{\Sigma}),$$

where  $\text{vec}(\mathbf{X})$  is a vector stacking the columns of  $\mathbf{X}$ , and  $\mathbf{V} \otimes \boldsymbol{\Sigma}$  denotes the Kronecker product.

## Author(s)

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## See Also

Useful links:

- <https://github.com/mlysy/mniw/>
- <https://mlysy.github.io/mniw/>
- Report bugs at <https://github.com/mlysy/mniw/issues>

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crossprodV	<i>Matrix cross-product.</i>
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## Description

Vectorized matrix cross-products  $t(X) \setminus V \setminus Y$  or  $t(X) \setminus V^{-1} \setminus Y$ .

## Usage

```
crossprodV(X, Y = NULL, V, inverse = FALSE)
```

## Arguments

X	A matrix of size $p \times q$ , or an array of size $p \times q \times n$ .
Y	A matrix of size $p \times r$ , or an array of size $p \times r \times n$ . If missing defaults to $Y = X$ .
V	A matrix of size $p \times p$ , or an array of size $p \times p \times n$ .
inverse	Logical; whether or not the inner product should be calculated with $V$ or $V^{-1}$ .

## Value

An array of size  $q \times r \times n$ .

## Examples

```
# problem dimensions
p <- 4
q <- 2
r <- 3
n <- 5
X <- array(rnorm(p*q*n), dim = c(p, q, n)) # vectorized
Y <- array(rnorm(p*r*n), dim = c(p, r, n)) # vectorized
V <- crossprod(matrix(rnorm(p*p), p, p)) # not vectorized (but positive definite)
crossprodV(X = X, V = V) # self cross-product
# cross-product with inverse matrix weight
crossprodV(X = X, V = V, Y = Y, inverse = TRUE)
```

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Hospitals	<i>Hospital profiling data.</i>
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## Description

Information on patient-reported problem rates for 27 teaching hospitals and private academic health centers in the United States.

## Usage

Hospitals

## Format

A data frame with 27 rows (one for each hospital) and 4 variables:

NSrg Non-surgery related problem rate (percent).

Srg Surgery related problem rate (percent).

Severity Average health index for surveyed patients.

Size Number of patients surveyed.

## References

Everson, P.J. and Morris, C.N. "Inference for multivariate normal hierarchical models." *Journal of the Royal Statistical Society, Series B* 62:2 (2000): 399-412.

MatrixNormal-dist      *The Matrix-Normal distribution.*

## Description

Density and random sampling for the Matrix-Normal distribution.

## Usage

`dMNorm(X, Lambda, SigmaR, SigmaC, log = FALSE)`

`rMNorm(n, Lambda, SigmaR, SigmaC)`

## Arguments

X	Argument to the density function. Either a $p \times q$ matrix or a $p \times q \times n$ array.
Lambda	Mean parameter. Either a $p \times q$ matrix or a $p \times q \times n$ array.
SigmaR	Between-row covariance matrix. Either a $p \times p$ matrix or a $p \times p \times n$ array.
SigmaC	Between-column covariance matrix. Either a $q \times q$ matrix or a $q \times q \times n$ array.
log	Logical; whether or not to compute the log-density.
n	Integer number of random samples to generate.

## Details

The Matrix-Normal distribution  $\mathbf{X} \sim \text{Matrix-Normal}(\boldsymbol{\Lambda}, \boldsymbol{\Sigma}_R, \boldsymbol{\Sigma}_C)$  on the random matrix  $\mathbf{X}_{p \times q}$  is defined as

$$\text{vec}(\mathbf{X}) \sim \mathcal{N}(\text{vec}(\boldsymbol{\Lambda}), \boldsymbol{\Sigma}_C \otimes \boldsymbol{\Sigma}_R),$$

where  $\text{vec}(\mathbf{X})$  is a vector stacking the columns of  $\mathbf{X}$ , and  $\boldsymbol{\Sigma}_C \otimes \boldsymbol{\Sigma}_R$  denotes the Kronecker product.

**Value**

A vector length n for density evaluation, or an array of size p x q x n for random sampling.

**Examples**

```
# problem dimensions
p <- 4
q <- 2
n <- 10 # number of observations
# parameter values
Lambda <- matrix(rnorm(p*q),p,q) # mean matrix
# row-wise variance matrix (positive definite)
SigmaR <- crossprod(matrix(rnorm(p*p), p, p))
SigmaC <- rwish(n, Psi = diag(q), nu = q + 1) # column-wise variance (vectorized)

# random sample
X <- rMNorm(n, Lambda = Lambda, SigmaR = SigmaR, SigmaC = SigmaC)

# log-density at each sampled value
dMNorm(X, Lambda = Lambda, SigmaR = SigmaR, SigmaC = SigmaC, log = TRUE)
```

**MatrixT-dist**

*The Matrix-t distribution.*

**Description**

Density and sampling for the Matrix-t distribution.

**Usage**

dMT(X, Lambda, SigmaR, SigmaC, nu, log = FALSE)

rMT(n, Lambda, SigmaR, SigmaC, nu)

**Arguments**

X	Argument to the density function. Either a p x q matrix or a p x q x n array.
Lambda	Mean parameter. Either a p x q matrix or a p x q x n array.
SigmaR	Between-row covariance matrix. Either a p x p matrix or a p x p x n array.
SigmaC	Between-column covariance matrix. Either a q x q matrix or a q x q x n array.
nu	Degrees-of-freedom parameter. A scalar or vector of length n.
log	Logical; whether or not to compute the log-density.
n	Integer number of random samples to generate.

## Details

The Matrix-T distribution  $\mathbf{X} \sim \text{Matrix-T}(\boldsymbol{\Lambda}, \boldsymbol{\Sigma}, \boldsymbol{\Psi}, \nu)$  on a random matrix  $\mathbf{X}_{p \times q}$  is the marginal distribution of  $\mathbf{X}$  in  $(\mathbf{X}, \mathbf{V}) \sim \text{MNIW}(\boldsymbol{\Lambda}, \boldsymbol{\Sigma}, \boldsymbol{\Psi}, \nu)$ , where the Matrix-Normal Inverse-Wishart (MNIW) distribution is defined in [MNIW-dist](#).

## Value

A vector length  $n$  for density evaluation, or an array of size  $p \times q \times n$  for random sampling.

**MNIW-dist**

*Generate samples from the Matrix-Normal Inverse-Wishart distribution.*

## Description

Generate samples from the Matrix-Normal Inverse-Wishart distribution.

## Usage

```
rMNIW(n, Lambda, Sigma, Psi, nu, prec = FALSE)
```

```
rmniw(n, Lambda, Omega, Psi, nu)
```

## Arguments

<code>n</code>	number of samples.
<code>Lambda</code>	A mean matrix of size $p \times q$ or an array of size $p \times q \times n$ . Defaults to matrix of zeros when missing.
<code>Sigma</code>	A row-wise variance or precision matrix of size $p \times p$ , or an array of size $p \times p \times n$ . Defaults to the identity matrix when missing.
<code>Psi</code>	A scale matrix of size $q \times q$ , or an array of size $q \times q \times n$ . Defaults to identity matrix when missing.
<code>nu</code>	Scalar degrees-of-freedom parameter.
<code>prec</code>	Logical; whether or not <code>Sigma</code> is on the variance or precision scale.
<code>Omega</code>	A between-row precision matrix of size $p \times p$ , or an array of size $p \times p \times n$ . Defaults to the identity matrix when missing.

## Details

The Matrix-Normal Inverse-Wishart (MNIW) distribution  $(\mathbf{X}, \mathbf{V}) \sim \text{MNIW}(\boldsymbol{\Lambda}, \boldsymbol{\Sigma}, \boldsymbol{\Psi}, \nu)$  on random matrices  $\mathbf{X}_{p \times q}$  and symmetric positive-definite  $\mathbf{V}_{q \times q}$  is defined as

$$\begin{aligned} \mathbf{V} &\sim \text{Inverse-Wishart}(\boldsymbol{\Psi}, \nu) \\ \mathbf{X} | \mathbf{V} &\sim \text{Matrix-Normal}(\boldsymbol{\Lambda}, \boldsymbol{\Sigma}, \mathbf{V}), \end{aligned}$$

where the Matrix-Normal distribution is defined as the multivariate normal

$$\text{vec}(\mathbf{X}) \sim \mathcal{N}(\text{vec}(\boldsymbol{\Lambda}), \mathbf{V} \otimes \boldsymbol{\Sigma}),$$

where  $\text{vec}(\mathbf{X})$  is a vector stacking the columns of  $\mathbf{X}$ , and  $\mathbf{V} \otimes \boldsymbol{\Sigma}$  denotes the Kronecker product.

`rMNIW()` is a convenience wrapper to `rMNIW(Sigma = Omega, prec = TRUE)`, for the common situation in Bayesian inference with conjugate priors when between-row variances are naturally parametrized on the precision scale.

## Value

A list with elements:

- X Array of size p x q x n random samples from the Matrix-Normal component (see Details).
- V Array of size q x q x n of random samples from the Inverse-Wishart component.

## Examples

```
# problem dimensions
p <- 2
q <- 3
n <- 10 # number of samples
# parameter specification
Lambda <- matrix(rnorm(p*q),p,q) # single argument
Sigma <- rwish(n, Psi = diag(p), nu = p + rexp(1)) # vectorized argument
Psi <- rwish(n = 1, Psi = diag(q), nu = q + rexp(1)) # single argument
nu <- q + rexp(1)
# simulate n draws
rMNIW(n, Lambda = Lambda, Sigma = Sigma, Psi = Psi, nu = nu)
```

## Description

Density and random sampling for the Multivariate Normal distribution.

## Usage

```
dmNorm(x, mu, Sigma, log = FALSE)

rmNorm(n, mu, Sigma)
```

## Arguments

<code>x</code>	Argument to the density function. A vector of length $q$ or an $n \times q$ matrix.
<code>mu</code>	Mean vector(s). Either a vector of length $q$ or an $n \times q$ matrix. If missing defaults to a vector of zeros.
<code>Sigma</code>	Covariance matrix or matrices. Either a $q \times q$ matrix or a $q \times q \times n$ array. If missing defaults to the identity matrix.
<code>log</code>	Logical; whether or not to compute the log-density.
<code>n</code>	Integer number of random samples to generate.

## Value

A vector for densities, or a  $n \times q$  matrix for random sampling.

## Examples

```
# Parameter specification
q <- 4 # number of dimensions
mu <- 1:q # mean vector
V <- toeplitz(exp(-seq(1:q))) # variance matrix

# Random sample
n <- 100
X <- rmNorm(n, mu, V)

# Calculate log density for each sampled vector
dmNorm(X, mu, V, log = TRUE)
```

**rRxNorm**

*Conditional sampling for Multivariate-Normal Random-Effects model.*

## Description

Sample from the conditional parameter distribution given the data and hyperparameters of the Multivariate-Normal Random-Effects (mNormRE) model (see **Details**).

## Usage

```
rRxNorm(n, x, V, lambda, Sigma)
```

## Arguments

<code>n</code>	Integer number of random samples to generate.
<code>x</code>	Data observations. Either a vector of length $q$ or a $n \times q$ matrix. In the latter case each row is a different vector.
<code>V</code>	Observation variances. Either a matrix of size $q \times q$ or a $q \times q \times n$ array.

lambda	Prior means. Either a vector of length q or an n x q matrix. In the latter case each row is a different mean. Defaults to zeros.
Sigma	Prior variances. Either a matrix of size q x q or a q x q x n array. Defaults to identity matrix.

## Details

Consider the hierarchical multivariate normal model

$$\begin{aligned}\boldsymbol{\mu} &\sim \mathcal{N}(\boldsymbol{\lambda}, \boldsymbol{\Sigma}) \\ \mathbf{x} | \boldsymbol{\mu} &\sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{V}).\end{aligned}$$

The Multivariate-Normal Random-Effects model  $\boldsymbol{\mu} \sim \text{RxNorm}(\mathbf{x}, \mathbf{V}, \boldsymbol{\lambda}, \boldsymbol{\Sigma})$  on the random vector  $\boldsymbol{\mu}_q$  is defined as the posterior distribution  $p(\boldsymbol{\mu} | \mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\Sigma})$ . This distribution is multivariate normal; for the mathematical specification of its parameters please see `vignette("mniw-distributions", package = "mniw")`.

## Examples

```
# data specification
q <- 5
y <- rnorm(q)
V <- rwish(1, diag(q), q+1)
# prior specification
lambda <- rep(0,q)
A <- diag(q)
n <- 10
# random sampling
rRxNorm(n, y, V, lambda, A)
```

## Description

Gibbs sampler for posterior distribution of parameters and hyperparameters of a multivariate normal random-effects linear regression model called RxNormLM (see Details).

## Usage

```
RxNormLM(
  nsamples,
  Y,
  V,
  X,
  prior = NULL,
  init,
  burn,
  updateHyp = TRUE,
```

```

  storeHyp = TRUE,
  updateRX = TRUE,
  storeRX = FALSE
)

```

## Arguments

<code>nsamples</code>	number of posterior samples to draw.
<code>Y</code>	$N \times q$ matrix of responses.
<code>V</code>	Either a $q \times q$ variance matrix or an $q \times q \times N$ array of such matrices.
<code>X</code>	$N \times p$ matrix of covariates.
<code>prior</code>	parameters of the prior MNIW distribution on the hyperparameters (see Details).
<code>init</code>	(optional) list with elements <code>Beta</code> , <code>Sigma</code> , and <code>Mu</code> providing the initial values for these. Default values are <code>Beta = matrix(0, p, q)</code> , <code>Sigma = diag(q)</code> , and <code>Mu = Y</code> .
<code>burn</code>	integer number of burn-in samples, or fraction of <code>nsamples</code> to prepend as burn-in.
<code>updateHyp, storeHyp</code>	logical. Whether or not to update/store the hyperparameter draws.
<code>updateRX, storeRX</code>	logical. Whether or not to update/store the random-effects draws.

## Details

The RxNormLM model is given by

$$y_i | \mu_i \sim_i idN(\mu_i, V_i)$$

$$\mu_i | \beta, \Sigma \sim iindN(x_i' \beta, \Sigma)$$

$$\beta, \Sigma \sim MNIW(\Lambda, \Omega^{-1}, \Psi, \nu),$$

where  $y_i$  and  $\mu_i$  are response and random-effects vectors of length  $q$ ,  $x_i$  are covariate vectors of length  $p$ , and  $(\beta, \Sigma)$  are hyperparameter matrices of size  $p \times q$  and  $q \times q$ .

The MNIW prior distribution is given by a list with elements `Lambda`, `Omega`, `Psi`, and `nu`. If any of these is `NULL` or missing, the default value is 0. Note that `Omega == 0` gives a Lebesgue prior to  $\beta$ .

## Value

A list with (potential) elements:

`Beta` An  $p \times q \times nsamples$  array of regression coefficient iterations (if `storeHyp == TRUE`)

`Sigma` An  $q \times q \times nsamples$  array of regression variance matrices (if `storeHyp == TRUE`)

`Mu` An  $n \times q \times nsamples$  array of random effects (if `storeRX == TRUE`)

## Examples

```

# problem dimensions
n <- sample(10:20,1) # number of observations
p <- sample(1:4,1) # number of covariates
q <- sample(1:4,1) # number of responses

# hyperparameters
Lambda <- rMNorm(1, Lambda = matrix(0, p, q))
Omega <- crossprod(rMNorm(1, Lambda = matrix(0, p, p)))
Psi <- crossprod(rMNorm(1, Lambda = matrix(0, q, q)))
nu <- rexp(1) + (q+1)
prior <- list(Lambda = Lambda, Omega = Omega, Psi = Psi, nu = nu)

# random-effects parameters
BSig <- rmniw(1, Lambda = Lambda, Omega = Omega, Psi = Psi, nu = nu)
Beta <- BSig$X
Sigma <- BSig$V

# design matrix
X <- rMNorm(1, matrix(0, n, p))

# random-effects themselves
Mu <- rmNorm(n, X %*% Beta, Sigma)

# generate response data
V <- rwish(n, Psi = diag(q), nu = q+1) # error variances
Y <- rmNorm(n, mu = Mu, Sigma = V) # responses

# visual checks for each component of Gibbs sampler

# sample from p(Mu | Beta, Sigma, Y)
nsamples <- 1e5
out <- RxNormLM(nsamples,
                 Y = Y, V = V, X = X,
                 prior = prior,
                 init = list(Beta = Beta, Sigma = Sigma, Mu = Mu),
                 burn = floor(nsamples/10),
                 updateHyp = FALSE,
                 storeHyp = FALSE,
                 updateRX = TRUE,
                 storeRX = TRUE)

# conditional distribution is RxNorm:
iObs <- sample(n, 1) # pick an observation at random
# calculate the RxNorm parameters
G <- Sigma %*% solve(V[,,iObs] + Sigma)
xB <- c(X[iObs,,drop=FALSE] %*% Beta)
muRx <- G %*% (Y[iObs,] - xB) + xB
SigmaRx <- G %*% V[,,iObs]

# a' * mu_i is univariate normal with known mean and variance:
a <- rnorm(q) # arbitrary vector

```

```

amui <- crossprod(a, out$Mu[iObs,,]) # a' * mu_i

hist(amui, breaks = 100, freq = FALSE,
     xlab = "", main = expression("Histogram of " * a^T * mu[i]))
curve(dnorm(x, mean = sum(a * muRx),
             sd = sqrt(crossprod(a, SigmaRx %*% a)[1])),
       add = TRUE, col = "red")
legend("topright",
       legend = c("Observed", "Expected"),
       lwd = c(NA, 2), pch = c(22, NA), seg.len = 1.5,
       col = c("black", "red"), bg = c("white", NA))

# sample from p(Beta, Sigma | Mu, Y)
nsamples <- 1e5
out <- RxNormLM(nsamples,
                 Y = Y, V = V, X = X,
                 prior = prior,
                 init = list(Beta = Beta, Sigma = Sigma, Mu = Mu),
                 burn = floor(nsamples/10),
                 updateHyp = TRUE,
                 storeHyp = TRUE,
                 updateRX = FALSE,
                 storeRX = FALSE)

# conditional distribution is MNIW:
# calculate the MNIW parameters
OmegaHat <- crossprod(X) + Omega
LambdaHat <- solve(OmegaHat, crossprod(X, Mu) + Omega %*% Lambda)
PsiHat <- Psi + crossprod(Mu) + crossprod(Lambda, Omega %*% Lambda)
PsiHat <- PsiHat - crossprod(LambdaHat, OmegaHat %*% LambdaHat)
nuHat <- nu + n

# a' Sigma^{-1} a is chi^2 with known parameters:
a <- rnorm(q)
aSiga <- drop(crossprodV(a, V = out$Sigma, inverse = TRUE))
sigX <- crossprod(a, solve(PsiHat, a))[1]
hist(aSiga, breaks = 100, freq = FALSE,
      xlab = "", main = expression("Histogram of " * a^T * Sigma^{-1} * a))
curve(dchisq(x/sigX, df = nuHat)/sigX, add = TRUE, col = "red")
legend("topright",
       legend = c("Observed", "Expected"),
       lwd = c(NA, 2), pch = c(22, NA), seg.len = 1.5,
       col = c("black", "red"), bg = c("white", NA))

# a' Beta b is student-t with known parameters:
a <- rnorm(p)
b <- rnorm(q)
# vectorized calculations
aBetab <- crossprodV(X = aperm(out$Beta, c(2,1,3)),
                      Y = b, V = diag(q)) # Beta b

```

```

aBetab <- drop(crossprodV(X = a, Y = aBetab, V = diag(p))) # a' Beta b
# student-t parameters
muT <- crossprod(a, LambdaHat %*% b)[1]
nuT <- nuHat-q+1
sigmaT <- crossprodV(a, V = OmegaHat, inverse = TRUE)[1]
sigmaT <- sigmaT * crossprodV(b, V = PsiHat)[1]
sigmaT <- sqrt(sigmaT / nuT)

hist(aBetab, breaks = 100, freq = FALSE,
      xlab = "", main = expression("Histogram of " * a^T * Beta * a))
curve(dt((x-muT)/sigmaT, df = nuT)/sigmaT, add = TRUE, col = "red")
legend("topright",
       legend = c("Observed", "Expected"),
       lwd = c(NA, 2), pch = c(22, NA), seg.len = 1.5,
       col = c("black", "red"), bg = c("white", NA))

```

**Wishart-dist***Wishart and Inverse-Wishart distributions.***Description**

Densities and random sampling for the Wishart and Inverse-Wishart distributions.

**Usage**

```

dwish(X, Psi, nu, log = FALSE)

rwish(n, Psi, nu)

diwish(X, Psi, nu, log = FALSE)

riwish(n, Psi, nu)

dwishart(X, Psi, nu, inverse = FALSE, log = FALSE)

rwishart(n, Psi, nu, inverse = FALSE)

```

**Arguments**

X	Argument to the density function. Either a q × q matrix or a q × q × n array.
Psi	Scale parameter. Either a q × q matrix or a q × q × n array.
nu	Degrees-of-freedom parameter. A scalar or vector of length n.
log	Logical; whether or not to compute the log-density.
n	Integer number of random samples to generate.
inverse	Logical; whether or not to use the Inverse-Wishart distribution.

## Details

The Wishart distribution  $\mathbf{X} \sim \text{Wishart}(\boldsymbol{\Psi}, \nu)$  on a symmetric positive-definite random matrix  $\mathbf{X}$  of size  $q \times q$  has PDF

$$f(\mathbf{X} | \boldsymbol{\Psi}, \nu) = \frac{|\mathbf{X}|^{(\nu-q-1)/2} \exp \left\{ -\text{tr}(\boldsymbol{\Psi}^{-1} \mathbf{X})/2 \right\}}{2^{q\nu/2} |\boldsymbol{\Psi}|^{\nu/2} \Gamma_q(\frac{\nu}{2})},$$

where  $\Gamma_q(\alpha)$  is the multivariate gamma function,

$$\Gamma_q(\alpha) = \pi^{q(q-1)/4} \prod_{i=1}^q \Gamma(\alpha + (1-i)/2).$$

The Inverse-Wishart distribution  $\mathbf{X} \sim \text{Inverse-Wishart}(\boldsymbol{\Psi}, \nu)$  is defined as  $\mathbf{X}^{-1} \sim \text{Wishart}(\boldsymbol{\Psi}^{-1}, \nu)$ . `dewish()` and `diwish()` are convenience wrappers for `dewishart()`, and similarly `rwish()` and `riwish()` are wrappers for `rwishart()`.

## Value

A vector length  $n$  for density evaluation, or an array of size  $q \times q \times n$  for random sampling.

## Examples

```
# Random sampling

n <- 1e5
q <- 3
Psi1 <- crossprod(matrix(rnorm(q^2), q, q))
nu <- q + runif(1, 0, 5)

X1 <- rwish(n, Psi1, nu) # Wishart

# plot it
plot_fun <- function(X) {
  q <- dim(X)[1]
  par(mfrow = c(q, q))
  for(ii in 1:q) {
    for(jj in 1:q) {
      hist(X[ii, jj, ], breaks = 100, freq = FALSE,
            xlab = "", main = parse(text = paste0("X[", ii, jj, "]")))
    }
  }
}

plot_fun(X1)

# "vectorized" scale parameter
Psi2 <- 5 * Psi1
vPsi <- array(c(Psi1, Psi2), dim = c(q, q, n))
X2 <- rwish(n, Psi = vPsi, nu = nu)
plot_fun(X2)
```

```
# Inverse-Wishart
X3 <- riwish(n, Psi2, nu)
plot_fun(X3)

# log-density calculation for sampled values
par(mfrow = c(1,1))
hist(dwish(X2, vPsi, nu, log = TRUE),
     breaks = 100, freq = FALSE, xlab = "",
     main = expression("log-p"*(X[2]^* | "*list(Psi,nu))))
```

# Index

## \* datasets

Hospitals, 3

crossprodV, 3

diwish (Wishart-dist), 13

dMNorm (MatrixNormal-dist), 4

dmNorm (MultiNormal-dist), 7

dMT (MatrixT-dist), 5

dwish (Wishart-dist), 13

dwishart (Wishart-dist), 13

Hospitals, 3

MatrixNormal (MatrixNormal-dist), 4

MatrixNormal-dist, 4

MatrixT (MatrixT-dist), 5

MatrixT-dist, 5

MNIW (MNIW-dist), 6

mniw (mniw-package), 2

MNIW-dist, 6, 6

mniw-package, 2

MultiNormal (MultiNormal-dist), 7

MultiNormal-dist, 7

riwish (Wishart-dist), 13

rMNIW (MNIW-dist), 6

rmniw (MNIW-dist), 6

rMNorm (MatrixNormal-dist), 4

rmNorm (MultiNormal-dist), 7

rMT (MatrixT-dist), 5

rRxNorm, 8

rwish (Wishart-dist), 13

rwishart (Wishart-dist), 13

RxNormLM, 9

Wishart (Wishart-dist), 13

Wishart-dist, 13