

# Package ‘latentgraph’

October 13, 2022

**Type** Package

**Title** Graphical Models with Latent Variables

**Version** 1.1

**Date** 2020-11-29

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**Description** Three methods are provided to estimate graphical models with latent variables: (1) Jin, Y., Ning, Y., and Tan, K. M. (2020) (preprint available); (2) Chandrasekaran, V., Parrilo, P. A. & Willsky, A. S. (2012) <doi:10.1214/11-AOS949>; (3) Tan, K. M., Ning, Y., Witten, D. M. & Liu, H. (2016) <doi:10.1093/biomet/asw050>.

**License** GPL-3

**Imports** Rcpp, pracma, glmnet, MASS, stats

**LinkingTo** Rcpp, RcppArmadillo

**NeedsCompilation** yes

**Repository** CRAN

**Date/Publication** 2020-12-10 09:10:09 UTC

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corlatent

*Graphical Models with Latent Variables and Correlated Replicates***Description**

Estimate graphical models with latent variables and correlated replicates using the method in Jin et al. (2020).

**Usage**

```
corlatent(data, accuracy, n, R, p, lambda1, lambda2, lambda3, distribution = "Gaussian",
rule = "AND")
```

**Arguments**

data	data set. Can be a matrix, list, array, or data frame. If the data set is a matrix, it should have $nR$ rows and $p$ columns. This matrix is formed by stacking $n$ matrices, and each matrix has $R$ rows and $p$ columns. If the data set is a data frame, the dimension and structure are the same as the matrix. If the data set is an array, its dimension is $(R, p, n)$ . If the data set is a list, it should have $n$ elements and each element is a matrix with $R$ rows and $p$ columns.
accuracy	the threshold where algorithm stops. The algorithm stops when the difference between estimators of the $(k - 1)$ th iteration and the $k$ th iteration is smaller than the value of accuracy.
n	the number of observations.
R	the number of replicates for each observation.
p	the number of observed variables.
lambda1	tuning parameter that encourages estimated graph to be sparse.
lambda2	tuning parameter that models the effects of correlated replicates. Usually set to be equal to lambda1.
lambda3	tuning parameter that encourages the latent effect to be piecewise constants.
distribution	For a data set with Gaussian distribution, use "Gaussian"; For a data set with Ising distribution, use "Ising". Default is "Gaussian".
rule	rules to combine matrices that encode the conditional dependence relationships between sets of two observed variables. Options are "AND" and "OR". Default is "AND".

**Details**

The corlatent method has two assumptions. Assumption 1 states that the  $R$  replicates are assumed to follow a one-lag vector autoregressive model, conditioned on the latent variables. Assumption 2 states that the latent variables are piecewise constant across replicates. Based on these two assumptions, the method solve the following problem for  $1 \leq j \leq p$ .

$$\min_{\theta_{j,-j}, \alpha_j, \Delta_j} \left\{ -\frac{1}{nR} l(\theta_{j,-j}, \alpha_j, \Delta_j) + \lambda \|\theta_{j,-j}\|_1 + \beta \|\alpha_j\|_1 + \gamma \|(I_n \otimes C)\Delta_j\|_1 \right\},$$

where  $l(\theta_{j,-j}, \alpha_j, \Delta_j)$  is the log likelihood function,  $\theta_{j,-j}$  encodes the conditional dependence relationships between  $j$ th observed variable and the other observed variables,  $\alpha_j$  models the correlation among replicates,  $\Delta_j$  encodes the latent effect,  $\lambda, \beta, \gamma$  are the tuning parameters,  $I_n$  is an  $n$ -dimensional identity matrix and  $C$  is the discrete first derivative matrix where the  $i$ th and  $(i+1)$ th column of every  $i$ th row are -1 and 1, respectively. This method aims at modeling exponential family graphical models with correlated replicates and latent variables.

### Value

omega	a matrix that encodes the conditional dependence relationships between sets of two observed variables
theta	the adjacency matrix with 0 and 1 encoding conditional independence and dependence between sets of two observed variables, respectively
penalties	the penalty values

### References

Jin, Y., Ning, Y., and Tan, K. M. (2020), 'Exponential Family Graphical Models with Correlated Replicates and Unmeasured Confounders', preprint available.

### Examples

```
# Gaussian distribution with "AND" rule
n <- 20
R <- 10
p <- 5
l <- 2
s <- 2
seed <- 1

data <- generate_Gaussian(n, R, p, l, s, sparsityA = 0.95, sparsityobserved = 0.9,
sparsitylatent = 0.2, lwb = 0.3, upb = 0.3, seed)$X

result <- corlatent(data, accuracy = 1e-6, n, R, p, lambda1 = 0.1, lambda2 = 0.1,
lambda3 = 1e+5, distribution = "Gaussian", rule = "AND")
```

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generate\_Gaussian      *Generate a Gaussian distributed data set*

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### Description

This function will generate a Gaussian distributed data set with latent variables and correlated replicates.

### Usage

```
generate_Gaussian(n, R, p, l, s, sparsityA, sparsityobserved, sparsitylatent, lwb,
upb, seed)
```

**Arguments**

n	the number of observations.
R	the number of replicates.
p	the number of observed variables.
l	the number of latent variables.
s	latent effects are generated as $s$ piecewise constant across replicates. The number $s$ should be a factor of $R$ .
sparsityA	proportion of the number of zeros in the transition matrix $A$ . Must be a number from 0 to 1.
sparsityobserved	proportion of the number of zeros in the inverse covariance of the observed variables. Must be a number from 0 to 1.
sparsitylatent	proportion of the number of zeros in the inverse covariances among latent variables and between observed variables and latent variables. Must be a number from 0 to 1.
lwb	lower bound for the elements in the inverse covariance matrix.
upb	upper bound for the elements in the inverse covariance matrix.
seed	the seed for the random number generator.

**Details**

This function aims to generate a Gaussian distributed data set with latent variables and correlated replicates. For each observation, the latent variables are piecewise constant across replicates, and conditioned on the latent variables, the replicates follow a one-lag vector autoregressive model, where the transition matrix  $A$  is sparse with non-zero elements set equal to 0.3. The matrix  $\Sigma$  is the covariance matrix for the observed variables  $X$  and the latent variables  $U$ , and we partition  $\Sigma$  into matrices that quantify the relationships among the observed variables ( $\Sigma_{XX}$ ), between the observed variables and latent variables ( $\Sigma_{XU}$  or  $\Sigma_{UX}$ ), and of the latent variables ( $\Sigma_{UU}$ ). In general, the data is generated with:

$$X_{i1}|U_{i1} \sim N_p(\Sigma_{XU}\Sigma_{UU}^{-1}U_{i1}, \Sigma_{XX} - \Sigma_{XU}\Sigma_{UU}^{-1}\Sigma_{UX}),$$

$$X_{it}|X_{i(t-1)}, U_{it} \sim N_p(AX_{i(t-1)} + \Sigma_{XU}\Sigma_{UU}^{-1}U_{it}, \Sigma_{XX} - \Sigma_{XU}\Sigma_{UU}^{-1}\Sigma_{UX}),$$

where  $1 \leq i \leq n$  and  $1 \leq t \leq R$ .

**Value**

X	the generated data, which is a list with $n$ elements and each element is a matrix with $R$ rows and $p$ columns
truegraph	a matrix that encodes the conditional dependence relationships between sets of two observed variables

## References

Jin, Y., Ning, Y., and Tan, K. M. (2020), ‘Exponential Family Graphical Models with Correlated Replicates and Unmeasured Confounders’, preprint available.

## Examples

```
data <- generate_Gaussian(n = 50, R = 20, p = 30, l = 2, s = 2, sparsityA = 0.95,
sparsityobserved = 0.9, sparsitylatent = 0.2, lwb = 0.3, upb = 0.3, seed = 1)
```

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 Ivglasso

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*Estimate Gaussian Graphical Models with Latent Variables*


---

## Description

Estimate Gaussian graphical models with latent variables using the method in Chandrasekaran et al. (2012).

## Usage

```
Ivglasso(data, n, p, lambda1, lambda2, rule = "AND")
```

## Arguments

data	data set, can be a matrix or data frame with $n$ rows and $p$ columns.
n	the number of observations.
p	the number of observed variables.
lambda1	tuning parameter that encourages estimated graph to be sparse. Lambda1 is proportional to lambda2.
lambda2	tuning parameter that encourages the matrix $K_{O,H}(K_H)^{-1}K_{H,O}$ to be low rank, where $K_H$ and $K_{O,H}$ quantify the dependencies among the latent variables and between the observed variables and latent variables, respectively. The matrix $K_{O,H}(K_H)^{-1}K_{H,O}$ summarizes the impact of marginalization over the latent variables.
rule	rules to combine the inverse covariance matrices. Options are "AND" and "OR". Default is "AND".

## Details

The Ivglasso method assumes that all the variables, both observed and latent, are jointly Gaussian, and specifies the conditional distribution of observed variables on the latent variables by a graphical model. Under the high-dimensional setting, this method provides consistent estimators for the conditional graphical model of observed variables conditioned on latent variables.

**Value**

omega	a matrix that encodes the conditional dependence relationships between sets of two observed variables
theta	the adjacency matrix with 0 and 1 encoding conditional independence and dependence between sets of two observed variables, respectively
penalties	the penalty values

**References**

Chandrasekaran, V., Parrilo, P. A. & Willsky, A. S. (2012), ‘Latent variable graphical model selection via convex optimization’, *Ann. Statist.* 40(4), 1935–1967.

**Examples**

```
#Gaussian distribution with "AND" rule
n <- 50
R <- 20
p <- 30
l <- 2
s <- 2
data <- generate_Gaussian(n, R, p, l, s, sparsityA = 0.95, sparsityobserved = 0.9,
sparsitylatent = 0.2, lwb = 0.3, upb = 0.3, seed = 1)$X

result <- lvglasso(data, n, p, lambda1 = 0.222, lambda2 = 0.1*0.222, rule = "AND")
```

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semilattent

---

*Estimate Graphical Models with Latent Variables and Replicates*


---

**Description**

Estimate graphical models with latent variables and replicates using the method in Tan et al. (2016).

**Usage**

```
semilattent(data, n, R, p, lambda, distribution = "Gaussian", rule = "AND")
```

**Arguments**

data	data set. Can be a matrix, list, array, or data frame. If the data set is a matrix, it should have $nR$ rows and $p$ columns. This matrix is formed by stacking $n$ matrices, and each matrix has $R$ rows and $p$ columns. If the data set is a data frame, the dimension and structure are the same as the matrix. If the data set is an array, its dimension is $(R, p, n)$ . If the data set is a list, it should have $n$ elements and each element is a matrix with $R$ rows and $p$ columns.
n	the number of observations.
R	the number of replicates for each observation.

p	the number of observed variables.
lambda	tuning parameter that encourages estimated graph to be sparse.
distribution	For a data set with Gaussian distribution, use "Gaussian"; For a data set with Ising distribution, use "Ising". Default is "Gaussian".
rule	rules to combine matrices that encode the conditional dependence relationships between sets of two observed variables. Options are "AND" and "OR". Default is "AND".

### Details

The semil latent method has two assumptions. The first one states that the latent variables are constant across replicates. Assumption 2 states that given the latent variables, the replicates are mutually independent. With these two assumptions, the method seeks to solve the following problem for  $1 \leq j \leq p$ .

$$\min_{\beta_{j,O/j}} \{l_j(\beta_{j,O/j}) + \lambda \|\beta_{j,O/j}\|_1\},$$

where  $l_j(\beta_{j,O/j})$  is a nuisance-free loss function,  $\beta_{j,O/j}$  is a parameter that represents the conditional dependence relationships between  $j$ th observed variable and the other observed variables, and  $\lambda$  is a tuning parameter. This method aims at modeling semiparametric exponential family graphical model with latent variables and replicates.

### Value

omega	a matrix that encodes the conditional dependence relationships between sets of two observed variables
theta	the adjacency matrix with 0 and 1 encoding conditional independence and dependence between sets of two observed variables, respectively
penalty	the penalty value

### References

Tan, K. M., Ning, Y., Witten, D. M. & Liu, H. (2016), 'Replicates in high dimensions, with applications to latent variable graphical models', *Biometrika* 103(4), 761–777.

### Examples

```
#semil latent Gaussian with "AND" rule
n <- 50
R <- 20
p <- 30
seed <- 1
l <- 2
s <- 2
data <- generate_Gaussian(n, R, p, l, s, sparsityA = 0.95, sparsityobserved = 0.9,
sparsitylatent = 0.2, lwb = 0.3, upb = 0.3, seed)$X

result <- semil latent(data, n, R, p, lambda = 0.1, distribution = "Gaussian",
rule = "AND")
```

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