# Package 'flashier'

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Type Package Date 2023-10-16 Title Empirical Bayes Matrix Factorization Version 1.0.7 URL https://github.com/willwerscheid/flashier BugReports https://github.com/willwerscheid/flashier/issues Description Methods for matrix factorization based on Wang and Stephens (2021) <https://jmlr.org/papers/v22/20-589.html>. **Depends** R (>= 3.4), ebnm (>= 0.1-21), magrittr Imports Matrix, parallel, dplyr, stringr, tibble, tidyr, softImpute, irlba, ggplot2 Suggests ashr, cowplot, testthat, knitr, rmarkdown, RcppML, rsvd **License** BSD\_3\_clause + file LICENSE **Encoding** UTF-8 LazyData true RoxygenNote 7.2.3 VignetteBuilder knitr NeedsCompilation no Author Jason Willwerscheid [aut, cre], Peter Carbonetto [aut], Wei Wang [aut], Matthew Stephens [aut], Eric Weine [ctb], Gao Wang [ctb] Maintainer Jason Willwerscheid <jwillwer@providence.edu> **Repository** CRAN

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# R topics documented:

fitted.flash	3
fitted.flash_fit	3
flash	4
flash_add_intercept	8
flash_backfit	9
flash_clear_timeout	10
flash_conv_crit_elbo_diff	11
	12
flash_conv_crit_max_chg_F 1	12
flash_conv_crit_max_chg_L	13
flash_ebnm	14
flash_factors_fix	16
flash_factors_init	17
flash_factors_remove	18
flash_factors_reorder	19
flash_factors_set_to_zero	19
	20
flash_fit	20
flash_greedy	22
flash_greedy_init_default	24
flash_greedy_init_irlba	26
flash_greedy_init_softImpute	26
	27
flash_nullcheck	28
flash_set_conv_crit	29
flash_set_timeout	30
flash_set_verbose	31
flash_verbose_elbo	33
	34
flash_verbose_max_chg	35
flash_verbose_max_chg_F	35
flash_verbose_max_chg_L	36
gtex	37
gtex_colors	38
ldf	38
plot.flash	39
residuals.flash	11
residuals.flash_fit	41

Index

fitted.flash

# Description

Given a flash object, returns the "fitted values" E(LF') = E(L)E(F)'.

## Usage

## S3 method for class 'flash'
fitted(object, ...)

# Arguments

object	An object inheriting from class flash.
	Additional parameters are ignored.

## Value

The matrix of "fitted values."

fitted.flash\_fit Fitted method for flash fit objects

# Description

Given a flash\_fit object, returns the "fitted values" E(LF') = E(L)E(F)'.

# Usage

```
## S3 method for class 'flash_fit'
fitted(object, ...)
```

# Arguments

object	An object inheriting from class flash_fit.
	Additional parameters are ignored.

# Value

The matrix of "fitted values."

## Description

Fits an empirical Bayes matrix factorization (see **Details** for a description of the model). The resulting fit is referred to as a "flash" object (short for Factors and Loadings using Adaptive SHrinkage). Two interfaces are provided. The flash function provides a simple interface that allows a flash object to be fit in a single pass, while flash\_xxx functions are pipeable functions that allow for more complex flash objects to be fit incrementally (available functions are listed below under **See Also**). See the vignettes and **Examples** for usage.

### Usage

```
flash(
   data,
   S = NULL,
   ebnm_fn = ebnm_point_normal,
   var_type = 0L,
   greedy_Kmax = 50L,
   backfit = FALSE,
   nullcheck = TRUE,
   verbose = 1L
)
```

# Arguments

data	The observations. Usually a matrix, but can also be a sparse matrix of class Matrix or a low-rank matrix representation as returned by, for example, svd, irlba, rsvd, or softImpute (in general, any list that includes fields u, d, and v will be interpreted as a low-rank matrix representation).
S	The standard errors. Can be NULL (in which case all residual variance will be estimated) or a matrix, vector, or scalar. S should be a scalar if standard errors are identical across observations. It should be a vector if standard errors either vary across columns but are constant within any given row, or vary across rows but are constant within any given column (flash will use the length of the vector to determine whether the supplied values correspond to rows or columns; if the data matrix is square, then the sense must be specified using parameter S_dim in function flash_init).
ebnm_fn	The function or functions used to solve the empirical Bayes normal means (EBNM) subproblems. Most importantly, these functions specify the families of distributions $G_{\ell}^{(k)}$ and $G_{f}^{(k)}$ to which the priors on loadings and factors $g_{\ell}^{(k)}$ and $g_{f}^{(k)}$ are assumed to belong. If the same function is to be used for both loadings $L$ and factors $F$ , then ebnm_fn can be a single function. If one function is to be used for loadings and a second for factors, then ebnm_fn should be a list of length two, with the first element giving the function for loadings and the second

# flash

	<ul> <li>the function for factors. If different functions are to be used for different values of k, then factor/loadings pairs must be added successively using multiple calls to either flash_greedy or flash_factors_init.</li> <li>Any EBNM function provided by package ebnm can be used as input. Non-default arguments to parameters can be supplied using the helper function flash_ebnm. Custom EBNM functions can also be used: for details, see flash_ebnm.</li> </ul>
var_type	Describes the structure of the estimated residual variance. Can be NULL, 0, 1, 2, or $c(1, 2)$ . If NULL, then S accounts for all residual variance. If var_type = 0, then the estimated residual variance (which is added to any variance given by S) is assumed to be constant across all observations. Setting var_type = 1 estimates a single variance parameter for each row; var_type = 2 estimates one parameter for each column; and var_type = $c(1, 2)$ optimizes over all rank-one matrices (that is, it assumes that the residual variance parameter $s_{ij}$ can be written $s_{ij} = a_i b_j$ , where the <i>n</i> -vector <i>a</i> and the <i>p</i> -vector <i>b</i> are to be estimated). Note that if any portion of the residual variance is to be estimated, then it is usually faster to set S = NULL and to let flash estimate all of the residual variance. Further, var_type = $c(1, 2)$ is typically much slower than other options, so it should be used with care.
greedy_Kmax	The maximum number of factors to be added. This will not necessarily be the total number of factors added by flash, since factors are only added as long as they increase the variational lower bound on the log likelihood for the model.
backfit	A "greedy" fit is performed by adding up to greedy_Kmax factors, optimizing each newly added factor in one go without returning to optimize previously added factors. When backfit = TRUE, flash will additionally perform a final "backfit" where all factors are cyclically updated until convergence. The back- fitting procedure typically takes much longer than the greedy algorithm, but it also usually improves the final fit to a significant degree.
nullcheck	If nullcheck = TRUE, then flash will check that each factor in the final flash object improves the overall fit. Any factor that fails the check will be removed.
verbose	When and how to display progress updates. Set to $0$ for none, 1 for updates after a factor is added or a backfit is completed, 2 for additional notifications about the variational lower bound, and 3 for updates after every iteration. It is also possible to output a single tab-delimited table of values using function flash_set_verbose with verbose = -1.

# Details

If Y is an  $n \times p$  data matrix, then the rank-one empirical Bayes matrix factorization model is:

$$Y = \ell f' + E,$$

where  $\ell$  is an *n*-vector of **loadings**, *f* is a *p*-vector of **factors**, and *E* is an  $n \times p$  matrix of **residuals** (or "errors"). Additionally:

$$\begin{split} e_{ij} \sim N(0,s_{ij}^2) : i = 1,...,n; j = 1,...,p \\ \ell \sim g_\ell \in G_\ell \\ f \sim g_f \in G_f. \end{split}$$

The residual variance parameters  $s_{ij}^2$  are constrained to have a simple structure and are fit via maximum likelihood. (For example, one might assume that all standard errors are identical:  $s_{ij}^2 = s^2$  for some  $s^2$  and for all i, j). The functions  $g_\ell$  and  $g_f$  are assumed to belong to some families of priors  $G_\ell$  and  $G_f$  that are specified in advance, and are estimated via variational approximation.

The general rank-K empirical Bayes matrix factorization model is:

$$Y = LF' + E$$

or

$$y_{ij} = \sum_{k} \ell_{ik} f_{jk} + e_{ij} : i = 1, ..., n; j = 1, ..., p,$$

where L is now a matrix of loadings and F is a matrix of factors.

Separate priors  $g_{\ell}^{(k)}$  and  $g_{f}^{(k)}$  are estimated via empirical Bayes, and different prior families may be used for different values of k. In general, then:

$$e_{ij} \sim N(0, s_{ij}^2) : i = 1, ..., n; j = 1, ..., p$$
$$\ell_{ik} \sim g_{\ell}^{(k)} \in G_{\ell}^{(k)} : i = 1, ..., n; k = 1, ..., K$$
$$f_{ik} \sim g_{f}^{(k)} \in G_{f}^{(k)} : j = 1, ..., p; k = 1, ..., K$$

Typically,  $G_{\ell}^{(k)}$  and  $G_{f}^{(k)}$  will be closed under scaling, in which case  $\ell_k$  and  $f_k$  are only identifiable up to a scaling factor  $d_k$ . In other words, we can write:

$$Y = LDF' + E,$$

where D is a diagonal matrix with diagonal entries  $d_1, ..., d_K$ . The model can then be made identifiable by constraining the scale of  $\ell_k$  and  $f_k$  for k = 1, ..., K.

#### Value

A flash object. Contains elements:

- n\_factors The total number of factor/loadings pairs K in the fitted model.
- pve The proportion of variance explained by each factor/loadings pair. Since factors and loadings are not required to be orthogonal, this should be interpreted loosely: for example, the total proportion of variance explained could be larger than 1.
- elbo The variational lower bound achieved by the fitted model.
- residuals\_sd Estimated residual standard deviations (these include any variance component given as an argument to S).
- L\_pm, L\_psd, L\_lfsr Posterior means, standard deviations, and local false sign rates for loadings *L*.
- F\_pm, F\_psd, F\_lfsr Posterior means, standard deviations, and local false sign rates for factors *F*.
- L\_ghat The fitted priors on loadings  $\hat{g}_{\ell}^{(k)}$ .
- F\_ghat The fitted priors on factors  $\hat{g}_{f}^{(k)}$ .

- sampler A function that takes a single argument nsamp and returns nsamp samples from the posterior distributions for factors F and loadings L.
- flash\_fit A flash\_fit object. Used by flash when fitting is not performed all at once, but
  incrementally via calls to various flash\_xxx functions.

The following methods are available:

fitted.flash Returns the "fitted values" E(LF') = E(L)E(F)'.

- residuals.flash Returns the expected residuals Y E(LF') = Y E(L)E(F)'.
- ldf.flash Returns an LDF decomposition (see **Details** above), with columns of L and F scaled as specified by the user.

#### References

Wei Wang and Matthew Stephens (2021). "Empirical Bayes matrix factorization." *Journal of Machine Learning Research* 22, 1–40.

## See Also

flash\_init, flash\_greedy, flash\_backfit, and flash\_nullcheck. For more advanced functionality, see flash\_factors\_init, flash\_factors\_fix, flash\_factors\_set\_to\_zero, flash\_factors\_remove, flash\_set\_verbose, and flash\_set\_conv\_crit. For extracting useful data from flash objects, see fitted.flash, residuals.flash, and ldf.flash.

## Examples

data(gtex)

```
# Fit up to 3 factors and backfit.
fl <- flash(gtex, greedy_Kmax = 3L, backfit = TRUE)</pre>
# This is equivalent to the series of calls:
fl <- flash_init(gtex) %>%
 flash_greedy(Kmax = 3L) %>%
 flash_backfit() %>%
 flash_nullcheck()
# Fit a unimodal distribution with mean zero to each set of loadings
# and a scale mixture of normals with mean zero to each factor.
fl <- flash(gtex,</pre>
            ebnm_fn = c(ebnm_unimodal,
                        ebnm_normal_scale_mixture),
            greedy_Kmax = 3)
# Fit point-laplace priors using a non-default optimization method.
fl <- flash(gtex,</pre>
            ebnm_fn = flash_ebnm(prior_family = "point_laplace",
                                  optmethod = "trust"),
            greedy_Kmax = 3)
```

# Fit a "Kronecker" (rank-one) variance structure (this can be slow).

```
fl <- flash(gtex, var_type = c(1, 2), greedy_Kmax = 3L)</pre>
```

flash\_add\_intercept Add "intercept" to a flash object

#### Description

Adds an all-ones vector as a fixed set of loadings (if rowwise = TRUE) or fixed factor (if rowwise = FALSE). Assuming (without loss of generality) that the fixed factor/loadings is indexed as k = 1, a fixed set of loadings gives:

$$\mathbf{y}_{i\cdot} \approx \mathbf{f}_1 + \sum_{k=2}^{K} \ell_{ik} \mathbf{f}_k,$$

so that the (estimated) factor  $\mathbf{f}_1 \in \mathbf{R}^p$  is shared by all row-wise observations  $\mathbf{y}_{i} \in \mathbf{R}^p$ . A fixed factor gives:

$$\mathbf{y}_{\cdot j} \approx \boldsymbol{\ell}_1 + \sum_{k=2}^K f_{jk} \boldsymbol{\ell}_k,$$

so that the (estimated) set of loadings  $\ell_1 \in \mathbf{R}^n$  is shared by all column-wise observations  $y_{\cdot j} \in \mathbf{R}^n$ .

#### Usage

flash\_add\_intercept(flash, rowwise = TRUE, ebnm\_fn = ebnm\_point\_normal)

#### Arguments

flash	A flash or flash_fit object to which an "intercept" is to be added.
rowwise	Should the all-ones vector be added as a fixed set of loadings ("row-wise") or a fixed factor ("column-wise")? See above for details.
ebnm_fn	As with other factor/loadings pairs, a prior is put on the estimated factor (if rowwise = TRUE) or set of loadings (if rowwise = FALSE). Parameter ebnm_fn specifies the function used to estimate that prior; see flash for details.

#### Details

The estimated factor (if rowwise = TRUE) or set of loadings (if rowwise = FALSE) is initialized at the column- or row-wise means of the data (or, if factor/loadings pairs have previously been added, at the column- or row-wise means of the matrix of residuals) and then backfit via function flash\_backfit.

#### Value

The flash object from argument flash, with an "intercept" added.

8

# flash\_backfit

# Examples

flash\_backfit Backfit a flash object

# Description

Backfits existing flash factor/loadings pairs. Whereas a "greedy" fit optimizes each newly added factor/loadings pair in one go without returning to optimize previously added pairs, a "backfit" updates all existing pairs in a cyclical fashion. See flash for examples of usage.

## Usage

```
flash_backfit(
  flash,
  kset = NULL,
  extrapolate = TRUE,
  warmstart = TRUE,
  maxiter = 500,
  tol = NULL,
  verbose = NULL
)
```

# Arguments

flash kset	A flash or flash_fit object. A vector of integers specifying which factors to backfit. If kset = NULL, then all existing factors will be backfitted.
extrapolate	Whether to use an extrapolation technique inspired by Ang and Gillis (2019) to accelerate the fitting process. Control parameters are handled via global options and can be set by calling options ("extrapolate.control") <- control.param.
warmstart	Whether to use "warmstarts" when solving the EBNM subproblems by initializ- ing solutions at the previous value of the fitted prior $\hat{g}$ . An important side effect of warmstarts for ashr-like prior families is to fix the grid at its initial setting. Fixing the grid can lead to poor fits if there are large changes in the scale of the estimated prior over the course of the fitting process. However, allowing the grid to vary can occasionally result in decreases in ELBO.

maxiter	The maximum number of backfitting iterations. An "iteration" is defined such that all factors in kset get updated at each iteration.
tol	The convergence tolerance parameter. After each update, the fit is compared to the fit from before the update using a convergence criterion function (by default, the difference in ELBO, but the criterion can be changed via flash_set_conv_crit). The backfit is considered to have "converged" when the value of the convergence criterion function over successive updates to <i>all</i> factor/loadings pairs is less than or equal to tol. If, for example, factor/loadings pairs $1, \ldots, K$ are being se- quentially backfitted, then fits are compared before and after the update to fac- tor/loadings 1, before and after the update to factor/loadings 2, and so on through factor/loadings $K$ , and backfitting only terminates when the convergence crite- rion function returns a value less than or equal to tol for all $K$ updates. Note that specifying tol here will override any value set by flash_set_conv_crit; to use the "global" tolerance parameter, tol must be left unspecified (NULL). If tol = NULL and a global tolerance parameter has not been set, then the default tolerance used is $np\sqrt{\epsilon}$ , where $n$ is the number of rows in the dataset, $p$ is the number of columns, and $\epsilon$ is equal to .Machine\$double.eps.
verbose	When and how to display progress updates. Set to $0$ for none, 1 for updates after a factor is added or a backfit is completed, 2 for additional notifications about the variational lower bound, and 3 for updates after every iteration. It is also possible to output a single tab-delimited table of values using function flash_set_verbose with verbose = -1.

# Value

The flash object from argument flash, backfitted as specified.

flash\_clear\_timeout Set timeout

# Description

Used in a flash pipeline to clear timeout conditions set using flash\_set\_timeout.

# Usage

```
flash_clear_timeout(flash)
```

# Arguments

flash A flash or flash\_fit object.

# Value

The flash object from argument flash, with timeout settings cleared.

flash\_conv\_crit\_elbo\_diff

Calculate the difference in ELBO

# Description

The default objective function used to determine convergence when fitting a flash object. Calculates the difference in the variational lower bound ("ELBO") from one iteration to the next.

## Usage

flash\_conv\_crit\_elbo\_diff(curr, prev, k)

#### Arguments

curr	The flash_fit object from the current iteration.
prev	The flash_fit object from the previous iteration.
k	Only used during sequential backfits (that is, calls to flash_backfit where extrapolate = FALSE). It then takes the index of the factor/loadings pair currently being optimized.

## Details

This function is an example of a function that may be passed to parameter fn in function flash\_set\_conv\_crit to set the convergence criterion for a flash pipeline. See flash\_set\_conv\_crit for details and examples.

## Value

A scalar, which is compared against the tolerance parameter tol to determine whether a fit has converged.

## See Also

flash\_conv\_crit\_max\_chg flash\_conv\_crit\_max\_chg\_L, flash\_conv\_crit\_max\_chg\_F

```
flash_conv_crit_max_chg
```

Calculate the maximum absolute difference in scaled loadings and factors

## Description

An alternative objective function that can be used to determine convergence when fitting a flash object. Calculates the maximum (absolute) change over all (posterior expected values for) loadings  $\ell_{ik}$  and factors  $f_{jk}$ . At each iteration, the loadings vectors  $\ell_{.1}, \ldots, \ell_{.K}$  and factors  $f_{.1}, \ldots, f_{.K}$  are  $L^2$ -normalized.

#### Usage

```
flash_conv_crit_max_chg(curr, prev, k)
```

## Arguments

curr	The flash_fit object from the current iteration.
prev	The flash_fit object from the previous iteration.
k	Only used during sequential backfits (that is, calls to flash_backfit where extrapolate = FALSE). It then takes the index of the factor/loadings pair currently being optimized.

## Value

A scalar, which is compared against the tolerance parameter tol to determine whether a fit has converged.

#### See Also

flash\_conv\_crit\_elbo\_diff,flash\_conv\_crit\_max\_chg\_L flash\_conv\_crit\_max\_chg\_F

flash\_conv\_crit\_max\_chg\_F

Calculate the maximum absolute difference in scaled factors

#### Description

An alternative objective function that can be used to determine convergence when fitting a flash object. Calculates the maximum (absolute) change over all (posterior expected values for) factors  $f_{ik}$ . At each iteration, the factors  $f_{.1}, \ldots, f_{.K}$  are  $L^2$ -normalized.

#### Usage

flash\_conv\_crit\_max\_chg\_F(curr, prev, k)

#### Arguments

curr	The flash_fit object from the current iteration.
prev	The flash_fit object from the previous iteration.
k	Only used during sequential backfits (that is, calls to flash_backfit where extrapolate = FALSE). It then takes the index of the factor/loadings pair currently being optimized.

# Value

A scalar, which is compared against the tolerance parameter tol to determine whether a fit has converged.

## See Also

flash\_conv\_crit\_elbo\_diff,flash\_conv\_crit\_max\_chg flash\_conv\_crit\_max\_chg\_L

flash\_conv\_crit\_max\_chg\_L

Calculate the maximum absolute difference in scaled loadings

## Description

An alternative objective function that can be used to determine convergence when fitting a flash object. Calculates the maximum (absolute) change over all (posterior expected values for) loadings  $\ell_{ik}$ . At each iteration, the loadings vectors  $\ell_{\cdot 1}, \ldots, \ell_{\cdot K}$  are  $L^2$ -normalized.

## Usage

```
flash_conv_crit_max_chg_L(curr, prev, k)
```

#### Arguments

curr	The flash_fit object from the current iteration.
prev	The flash_fit object from the previous iteration.
k	Only used during sequential backfits (that is, calls to flash_backfit where extrapolate = FALSE). It then takes the index of the factor/loadings pair currently being optimized.

## Value

A scalar, which is compared against the tolerance parameter tol to determine whether a fit has converged.

## See Also

flash\_conv\_crit\_elbo\_diff,flash\_conv\_crit\_max\_chg flash\_conv\_crit\_max\_chg\_F

#### flash\_ebnm

#### Description

flash\_ebnm is a helper function that provides readable syntax for constructing ebnm functions that can serve as arguments to parameter ebnm\_fn in functions flash, flash\_greedy, and flash\_factors\_init (see **Examples** below). It is also possible to write a custom function from scratch: see **Details** below for a simple example. A more involved example can be found in the "Advanced flashier" vignette.

#### Usage

flash\_ebnm(...)

#### Arguments

• • •

Parameters to be passed to function ebnm in package ebnm. An argument to prior\_family should be provided unless the default family of point-normal priors is desired. Arguments to parameters x, s, or output must not be included. Finally, if g\_init is included, then fix\_g = TRUE must be as well. To fix a prior grid, use parameter scale rather than g\_init.

#### Details

As input to parameter ebnm\_fn in functions flash, flash\_greedy, and flash\_factors\_init, it should suffice for many purposes to provide functions from package ebnm as is (for example, one might set ebnm\_fn = ebnm\_point\_laplace). To use non-default arguments, function flash\_ebnm may be used (see **Examples**). Custom functions may also be written. In general, any function that is used as an argument to ebnm\_fn must accept parameters:

- s A vector of standard errors, or a scalar if all standard errors are equal.
- g\_init The prior g. Usually, this is left unspecified (NULL) and estimated from the data. If it is supplied and fix\_g = TRUE, then the prior is fixed at g\_init; if fix\_g = FALSE, then g\_init gives the initial value of g used during optimization.

In flashier, g is fixed during the wrap-up phase when estimating local false sign rates and constructing a sampler; and g\_init is used with fix\_g = FALSE to "warmstart" backfits (see flash\_backfit). If none of these features (local false sign rates, samplers, or warmstarts) are needed, then both g\_init and fix\_g can be ignored (the EBNM function must still accept them as parameters, but it need not do anything with their arguments).

- fix\_g If TRUE, the prior is fixed at g\_init instead of estimated. See the description of g\_init above.
- output A character vector indicating which values are to be returned. Custom EBNM functions can safely ignore this parameter (again, they must accept it as a parameter, but they do not need to do anything with its argument).

The return object must be a list that includes fields:

x A vector of observations.

- posterior A data frame that includes columns mean and second\_moment (the first and second moments for each posterior distribution  $p(\theta_i \mid s_i, \hat{g}), i = 1, ..., n$ ). Optionally, a column lfsr giving local false sign rates may also be included.
- fitted\_g The estimated prior  $\hat{g}$ . Within flashier, fitted\_g is only ever used as an argument to g\_init in subsequent calls to the same EBNM function, so the manner in which it is represented is unimportant.
- log\_likelihood The optimal log likelihood  $L(\hat{g}) := \sum_{i} \log p(x_i \mid \hat{g}, s_i).$
- posterior\_sampler An optional field containing a function that samples from the posterior distributions of the "means"  $\theta_i$ . If included, the function should take a single parameter nsamp and return a matrix where rows correspond to samples and columns correspond to observations (that is, there should be nsamp rows and *n* columns).

#### Value

A function that can be passed as argument to parameter ebnm\_fn in functions flash, flash\_greedy, and flash\_factors\_init.

#### See Also

ebnm

## Examples

```
# A custom EBNM function might be written as follows:
my_ebnm_fn <- function(x, s, g_init, fix_g, output) {</pre>
  ebnm_res <- ebnm_point_laplace(</pre>
    x = x,
    s = s,
    g_init = g_init,
    fix_g = fix_g,
    output = output,
    control = list(iterlim = 10)
  )
  return(ebnm_res)
}
# The following are equivalent:
fl1 <- flash(</pre>
  gtex,
  ebnm_fn = my_ebnm_fn,
  greedy_Kmax = 2
)
fl2 <- flash(</pre>
  gtex,
  ebnm_fn = flash_ebnm(
    prior_family = "point_laplace",
    control = list(iterlim = 10)
  ),
  greedy_Kmax = 2
)
```

# Description

Fixes loadings  $\ell_{\cdot k}$  or factors  $f_{\cdot k}$  for one or more factor/loadings pairs, so that their values are not updated during subsequent backfits. For a given pair, either the loadings or factor can be fixed, but not both, and either all entries or a subset can be fixed. To unfix, use function flash\_factors\_unfix. See flash\_factors\_init for an example of usage.

## Usage

```
flash_factors_fix(
  flash,
  kset,
  which_dim = c("factors", "loadings"),
  fixed_idx = NULL,
  use_fixed_in_ebnm = NULL
)
```

#### Arguments

flash	A flash or flash_fit object.
kset	A vector of integers indexing the factor/loadings pairs whose loadings or factors are to be fixed.
which_dim	Whether to fix factors or loadings.
fixed_idx	If fixed_idx = NULL, then all loadings or factor values will be fixed. If only a subset are to be fixed, then fixed_idx should be an appropriately-sized vector or matrix of values that can be coerced to logical. For example, if a subset of loadings for two factor/loadings pairs are to be fixed, then fixed_idx should be a length- $n$ vector or an $n$ by 2 matrix (where $n$ is the number of rows in the data matrix).
use_fixed_in_ebnm	

By default, fixed elements are ignored when solving the EBNM subproblem in order to estimate the prior  $\hat{g}$ . This behavior can be changed by setting use\_fixed\_in\_ebnm = TRUE. This is a global setting which applies to all factor/loadings pairs; behavior cannot vary from one factor/loadings pair to another.

## Value

The flash object from argument flash, with factors or loadings fixed as specified.

flash\_factors\_init Initialize flash factors at specified values

# Description

Initializes factor/loadings pairs at values specified by init. This function has two primary uses: 1. One can initialize multiple factor/loadings pairs at once using an SVD-like function and then optimize them via function flash\_backfit. Sometimes this results in a better fit than adding them one at a time via flash\_greedy. 2. One can initialize factor/loadings pairs and then fix the factor (or loadings) via function flash\_factors\_fix to incorporate "known" factors into a flash object. See below for examples of both use cases.

## Usage

flash\_factors\_init(flash, init, ebnm\_fn = ebnm\_point\_normal)

## Arguments

flash	A flash or flash_fit object to which factors are to be added.
init	An SVD-like object (specifically, a list containing fields u, d, and v), a flash or flash_fit object, or a list of matrices specifying the values at which factors and loadings are to be initialized (for a data matrix of size $n \times p$ , this should be a list of length two, with the first element a matrix of size $n \times k$ and the second a matrix of size $p \times k$ ). If a flash fit is supplied, then it will be used to initialize both the first and second moments of posteriors on loadings and factors. Otherwise, the supplied values will be used to initialize posterior means, with posterior second moments initialized as the squared values of the first moments. Missing entries are not allowed.
ebnm_fn	The function or functions used to solve the empirical Bayes normal means (EBNM) subproblems. Most importantly, these functions specify the families of distributions $G_{\ell}^{(k)}$ and $G_{f}^{(k)}$ to which the priors on loadings and factors $g_{\ell}^{(k)}$ and $g_{f}^{(k)}$ are assumed to belong. If the same function is to be used for both loadings $L$ and factors $F$ , then ebnm_fn can be a single function. If one function is to be used for loadings and a second for factors, then ebnm_fn should be a list of length two, with the first element giving the function for loadings and the second the function for factors. If different functions are to be used for different values of $k$ , then factor/loadings pairs must be added successively using multiple calls to either flash_greedy or flash_factors_init. Any EBNM function provided by package ebnm can be used as input. Nondefault arguments to parameters can be supplied using the helper function flash_ebnm.

#### Value

The flash object from argument flash, with factors and loadings initialized as specified.

## Examples

```
# Initialize several factors at once and backfit.
fl <- flash_init(gtex) %>%
  flash_factors_init(init = svd(gtex, nu = 5, nv = 5)) %>%
  flash_backfit()
# Add fixed loadings with \ell_i identically equal to one. This can be
#
   interpreted as giving a "mean" factor that accounts for different
#
   row-wise means.
ones <- matrix(1, nrow = nrow(gtex), ncol = 1)</pre>
# Initialize the factor at the least squares solution.
ls_soln <- t(solve(crossprod(ones), crossprod(ones, gtex)))</pre>
fl <- flash_init(gtex) %>%
  flash_factors_init(init = list(ones, ls_soln)) %>%
  flash_factors_fix(kset = 1, which_dim = "loadings") %>%
  flash_backfit() %>%
  flash_greedy(Kmax = 5L)
```

flash\_factors\_remove Remove factors from a flash object

## Description

Sets factor/loadings pairs to zero and then removes them from the flash object. Note that this will change the indices of existing pairs.

## Usage

```
flash_factors_remove(flash, kset)
```

## Arguments

flash	A flash or flash_fit object.
kset	A vector of integers specifying which factor/loadings pairs to remove.

## Value

The flash object from argument flash, with the factors specified by kset removed.

#### See Also

flash\_factors\_set\_to\_zero

18

flash\_factors\_reorder Reorder factors in a flash object

#### Description

Reorders the factor/loadings pairs in a flash object.

#### Usage

flash\_factors\_reorder(flash, kset)

# Arguments

flash	A flash or flash_fit object.
kset	A vector of integers specifying the new order of the factor/loadings pairs. All ex-
	isting factors must be included in kset; to drop factors, use flash_factors_remove.

# Value

The flash object from argument flash, with the factors reordered according to argument kset.

```
flash_factors_set_to_zero
```

Set flash factors to zero

#### Description

Sets factor/loadings pairs to zero but does not remove them from the flash object (so as to keep the indices of existing pairs the same).

# Usage

flash\_factors\_set\_to\_zero(flash, kset)

# Arguments

flash	A flash or flash_fit object.
kset	A vector of integers specifying which factor/loadings pairs to set to zero.

# Value

The flash object from argument flash, with the factors specified by kset set to zero.

## See Also

flash\_factors\_remove

flash\_factors\_unfix Unfix flash factors

#### Description

If loadings  $\ell_k$  or factors  $f_k$  for one or more factor/loadings pairs have been "fixed" using function flash\_factors\_fix, then they can be unfixed using function flash\_factors\_unfix.

## Usage

flash\_factors\_unfix(flash, kset)

#### Arguments

flash	A flash or flash_fit object.
kset	A vector of integers indexing the factor/loadings pairs whose values are to be unfixed.

## Value

The flash object from argument flash, with values for the factor/loadings pairs specified by kset unfixed.

flash\_fit

Extract a flash\_fit object

#### Description

flash\_fit objects are the "internal" objects used by flash functions to fit an EBMF model. Whereas flash objects (the end results of the fitting process) include user-friendly fields and methods, flash\_fit objects were not designed for public consumption and can be unwieldy. Nonetheless, some advanced flash functionality requires the wielding of flash\_fit objects. In particular, initialization, convergence, and "verbose" display functions all take one or more flash\_fit objects as input (see parameter init\_fn in function flash\_greedy; parameter fn in flash\_set\_conv\_crit; and parameter fns in flash\_set\_verbose). For users who would like to write custom functions, the accessor functions and methods enumerated below may prove useful. See flash\_set\_verbose for an example.

## Usage

flash\_fit(flash)

flash\_fit\_get\_pm(f, n)

flash\_fit\_get\_p2m(f, n)

flash\_fit\_get\_est\_tau(f)

flash\_fit\_get\_fixed\_tau(f)

flash\_fit\_get\_tau(f)

flash\_fit\_get\_elbo(f)

flash\_fit\_get\_KL(f, n)

flash\_fit\_get\_g(f, n)

## Arguments

flash	A flash object.
f	A flash_fit object.
n	Set $n = 1$ to access loadings L and $n = 2$ to access factors F).

# Details

The following S3 methods are available for flash\_fit objects at all times except while optimizing new factor/loadings pairs as part of a "greedy" fit:

fitted.flash\_fit Returns the "fitted values" E(LF') = E(L)E(F)'.

residuals.flash\_fit Returns the expected residuals Y - E(LF') = Y - E(L)E(F)'.

ldf.flash\_fit Returns an LDF decomposition, with columns of L and F scaled as specified by the user.

#### Value

See function descriptions below.

## Functions

- flash\_fit\_get\_pm(): The posterior means for the loadings matrix L (when parameter n is equal to 1) or factor matrix F (when n = 2). While optimizing new factor/loadings pairs as part of a "greedy" fit, only the posterior means for the new loadings  $\ell_{\cdot k}$  or factor  $f_{\cdot k}$  will be returned.
- flash\_fit\_get\_p2m(): The posterior second moments for the loadings matrix L (when parameter n is equal to 1) or factor matrix F (when n = 2). While optimizing new factor/loadings pairs, only the posterior second moments for the new loadings  $\ell_{\cdot k}$  or factor  $f_{\cdot k}$  will be returned.
- flash\_fit\_get\_est\_tau(): Equal to  $1/\sigma^2$ , where  $\sigma^2$  is the estimated portion of the residual variance (total, by row, or by column, depending on the variance type).
- flash\_fit\_get\_fixed\_tau(): Equal to  $1/s^2$ , where  $s^2$  is the fixed portion of the residual variance (total, by row, or by column).
- flash\_fit\_get\_tau(): The overall precision  $1/(\sigma^2 + s^2)$ .

- flash\_fit\_get\_elbo(): The variational lower bound (ELBO).
- flash\_fit\_get\_KL(): A vector containing the KL-divergence portions of the ELBO, with one value for each factor (when n = 2) or set of loadings (when n = 1). While optimizing new factor/loadings pairs, only the KL-divergence for the new factor or loadings will be returned.
- flash\_fit\_get\_g(): A list containing estimated priors on loadings ĝ<sub>ℓ</sub> (when n = 1) or factors ĝ<sub>f</sub> (when n = 2). While optimizing new factor/loadings pairs, only the estimated prior on the new factor or loadings will be returned.

flash\_greedy

Greedily add factors to a flash object

#### Description

Adds factor/loadings pairs to a flash object in a "greedy" manner. Up to Kmax pairs are added one at a time. At each step, flash\_greedy attempts to find an optimal additional (rank-one) factor given all previously added factors. The additional factor is retained if it increases the variational lower bound (ELBO); otherwise, fitting terminates. See flash for examples of usage.

#### Usage

```
flash_greedy(
  flash,
  Kmax = 1,
  ebnm_fn = ebnm_point_normal,
  init_fn = NULL,
  extrapolate = FALSE,
  warmstart = FALSE,
  maxiter = 500,
  tol = NULL,
  verbose = NULL
)
```

#### Arguments

flash	A flash or flash_fit object to which factors are to be added.
Kmax	The maximum number of factors to be added. This will not necessarily be the total number of factors added by flash_greedy, since factors are only added as long as they increase the ELBO.
ebnm_fn	The function or functions used to solve the empirical Bayes normal means (EBNM) subproblems. Most importantly, these functions specify the families of distributions $G_{\ell}^{(k)}$ and $G_{f}^{(k)}$ to which the priors on loadings and factors $g_{\ell}^{(k)}$
	and $g_f^{(k)}$ are assumed to belong. If the same function is to be used for both load- ings $L$ and factors $F$ , then ebnm_fn can be a single function. If one function is to be used for loadings and a second for factors, then ebnm_fn should be a list of length two, with the first element giving the function for loadings and the second

	<ul> <li>the function for factors. If different functions are to be used for different values of k, then factor/loadings pairs must be added successively using multiple calls to either flash_greedy or flash_factors_init.</li> <li>Any EBNM function provided by package ebnm can be used as input. Non-default arguments to parameters can be supplied using the helper function flash_ebnm.</li> <li>Custom EBNM functions can also be used: for details, see flash_ebnm.</li> </ul>
init_fn	The function used to initialize factor/loadings pairs. Functions flash_greedy_init_default, flash_greedy_init_softImpute, and flash_greedy_init_irlba have been supplied; note, in particular, that flash_greedy_init_softImpute can yield better results than the default initialization function when there is missing data. Custom initialization functions may also be used. If init_fn = NULL then flash_greedy_init_default will be used, with an attempt made to set argument sign_constraints appro- priately via test calls to the EBNM function(s) specified by parameter ebnm_fn. If factors or loadings are constrained in some other fashion (e.g., bounded sup- port), then the initialization function should be modified to account for the con- straints — otherwise, the greedy algorithm can stop adding factor/loadings pairs too early. Custom initialization functions should accept a single parameter re- ferring to a flash_fit object and should output a list consisting of two vectors, which will be used as initial values for the new loadings $\ell_{.k}$ and the new factor $f_{.k}$ . Typically, a custom initialization function will extract the matrix of resid- uals from the flash_fit object using method residuals.flash_fit and then return a (possibly constrained) rank-one approximation to the matrix of residu- als. See <b>Examples</b> below.
extrapolate	Whether to use an extrapolation technique inspired by Ang and Gillis (2019) to accelerate the fitting process. Control parameters are handled via global options and can be set by calling options("extrapolate.control") <- control.param.
warmstart	Whether to use "warmstarts" when solving the EBNM subproblems by initializ- ing solutions at the previous value of the fitted prior $\hat{g}$ . An important side effect of warmstarts for ashr-like prior families is to fix the grid at its initial setting. Fixing the grid can lead to poor fits if there are large changes in the scale of the estimated prior over the course of the fitting process. However, allowing the grid to vary can occasionally result in decreases in ELBO.
maxiter	The maximum number of iterations when optimizing a greedily added factor/loadings pair.
tol	The convergence tolerance parameter. At each iteration, the fit is compared to the fit from the previous iteration using a convergence criterion function (by de- fault, the difference in ELBO, but the criterion can be changed via flash_set_conv_crit). When the value returned by this function is less than or equal to tol, the newly added factor/loadings pair is considered to have "converged," so that flash_greedy moves on and attempts to add another new factor (or, if the maximum number of factors Kmax has been reached, the process terminates). Note that specifying tol here will override any value set by flash_set_conv_crit; to use the "global" tolerance parameter, tol must be left unspecified (NULL). If tol = NULL and a global tolerance parameter has not been set, then the default tolerance used is $np\sqrt{\epsilon}$ , where n is the number of rows in the dataset, p is the number of columns, and $\epsilon$ is equal to .Machine\$double.eps.

verbose When and how to display progress updates. Set to  $\emptyset$  for none, 1 for updates after a factor is added or a backfit is completed, 2 for additional notifications about the variational lower bound, and 3 for updates after every iteration. It is also possible to output a single tab-delimited table of values using function flash\_set\_verbose with verbose = -1.

#### Value

The flash object from argument flash, with up to Kmax new factor/loadings pairs "greedily" added.

## See Also

flash\_greedy\_init\_default, flash\_greedy\_init\_softImpute, flash\_greedy\_init\_irlba

#### Examples

```
# The following are examples of advanced usage. See ?flash for basic usage.
```

```
# Increase the maximum number of iterations in the default initialization
   method.
#
my_init_fn <- function(f) flash_greedy_init_default(f, maxiter = 500)</pre>
fl <- flash_init(gtex) %>%
  flash_greedy(init_fn = my_init_fn)
# Use a custom initialization function that wraps function nmf from
#
    package RcppML.
nmf_init_fn <- function(f) {</pre>
  nmf_res <- RcppML::nmf(resid(f), k = 1, verbose = FALSE)</pre>
  return(list(as.vector(nmf_res$w), as.vector(nmf_res$h)))
}
fl.nmf <- flash_init(gtex) %>%
  flash_greedy(ebnm_fn = ebnm_unimodal_nonnegative,
               init_fn = nmf_init_fn)
```

# Description

The default method for initializing the loadings  $\ell_{\cdot k}$  and factor values  $f_{\cdot k}$  of a new ("greedy") flash factor. It is essentially an implementation of the power method, but unlike many existing implementations, it can handle missing data and sign constraints. For details, see Chapter 2.2.3 in the reference below.

# Usage

```
flash_greedy_init_default(
  flash,
  sign_constraints = NULL,
  tol = NULL,
  maxiter = 100,
  seed = 666
)
```

# Arguments

flash	A flash_fit obje	ect.

sign\_constraints

	This parameter can be used to constrain the sign of the initial factor and loadings. It should be a vector of length two with entries equal to -1, 0, or 1. The first entry constrains the sign of the loadings $\ell_{\cdot k}$ , with -1 yielding nonpositive loadings, +1 yielding nonnegative loadings, and 0 indicating that loadings should not be constrained. The second entry of sign_constraints similarly constraints the sign of factor values $f_{\cdot k}$ . If sign_constraints = NULL, then no constraints will be applied.
tol	Convergence tolerance parameter. When the maximum (absolute) change over all values $\ell_{ik}$ and $f_{jk}$ is less than or equal to tol, initialization terminates. At each iteration, the factor and loadings are $L^2$ -normalized. The default tolerance parameter is $\min(1/n, 1/p)$ , where n is the number of rows in the data matrix and p is the number of columns.
maxiter	Maximum number of power iterations.
seed	Since initialization is random, a default seed is set for reproducibility.

# Value

A list of length two consisting of, respectively, the vector of initial values for loadings  $\ell_{\cdot k}$  and the vector of initial factor values  $f_{\cdot k}$ .

# References

Jason Willwerscheid (2021), *Empirical Bayes Matrix Factorization: Methods and Applications*. Ph.D. thesis, University of Chicago.

## See Also

flash\_greedy, flash\_greedy\_init\_softImpute, flash\_greedy\_init\_irlba

flash\_greedy\_init\_irlba

Initialize a flash factor using IRLBA

# Description

Initializes a new ("greedy") flash factor using irlba. This can be somewhat faster than flash\_greedy\_init\_default for large, dense data matrices. For sparse matrices of class Matrix, the default initialization should generally be preferred.

## Usage

flash\_greedy\_init\_irlba(flash, seed = 666, ...)

## Arguments

flash	A flash_fit object.
seed	Since initialization is random, a default seed is set for reproducibility.
	Additional parameters to be passed to irlba.

#### Value

A list of length two consisting of, respectively, the vector of initial values for loadings  $\ell_{\cdot k}$  and the vector of initial factor values  $f_{\cdot k}$ .

#### See Also

flash\_greedy, flash\_greedy\_init\_default, flash\_greedy\_init\_softImpute

## Description

Initializes a new ("greedy") flash factor using softImpute. When there is missing data, this can yield better results than flash\_greedy\_init\_default without sacrificing much (if any) speed.

## Usage

```
flash_greedy_init_softImpute(flash, seed = 666, ...)
```

# flash\_init

#### Arguments

flash	A flash_fit object.
seed	Since initialization is random, a default seed is set for reproducibility.
	Additional parameters to be passed to softImpute.

# Value

A list of length two consisting of, respectively, the vector of initial values for loadings  $\ell_{k}$  and the vector of initial factor values  $f_{k}$ .

## See Also

flash\_greedy,flash\_greedy\_init\_default,flash\_greedy\_init\_irlba

flash_init	Initialize flash object	
------------	-------------------------	--

# Description

Sets up a flash object with no factors. Since all other flash\_xxx functions take a flash or flash\_fit object as their first argument, calling flash\_init should be the first step in any flash pipeline. See flash for examples of usage.

## Usage

flash\_init(data, S = NULL, var\_type = 0L, S\_dim = NULL)

# Arguments

data	The observations. Usually a matrix, but can also be a sparse matrix of class Matrix or a low-rank matrix representation as returned by, for example, svd, irlba, rsvd, or softImpute (in general, any list that includes fields u, d, and v will be interpreted as a low-rank matrix representation).
S	The standard errors. Can be NULL (in which case all residual variance will be estimated) or a matrix, vector, or scalar. S should be a scalar if standard errors are identical across observations. It should be a vector if standard errors either vary across columns but are constant within any given row, or vary across rows but are constant within any given column (flash will use the length of the vector to determine whether the supplied values correspond to rows or columns; if the data matrix is square, then the sense must be specified using parameter S_dim in function flash_init).
var_type	Describes the structure of the estimated residual variance. Can be NULL, 0, 1, 2, or $c(1, 2)$ . If NULL, then S accounts for all residual variance. If var_type = 0, then the estimated residual variance (which is added to any variance given by S) is assumed to be constant across all observations. Setting var_type = 1 estimates a single variance parameter for each row; var_type = 2 estimates one

	parameter for each column; and var_type = $c(1, 2)$ optimizes over all rank- one matrices (that is, it assumes that the residual variance parameter $s_{ij}$ can be written $s_{ij} = a_i b_j$ , where the <i>n</i> -vector <i>a</i> and the <i>p</i> -vector <i>b</i> are to be estimated). Note that if any portion of the residual variance is to be estimated, then it is usu- ally faster to set S = NULL and to let flash estimate all of the residual variance. Further, var_type = $c(1, 2)$ is typically much slower than other options, so it should be used with care.
S_dim	If the argument to S is a vector and the data matrix is square, then S_dim must specify whether S encodes row-wise or column-wise standard errors. More precisely, if S_dim = 1, then S will be interpreted as giving standard errors that vary across rows but are constant within any particular row; if S_dim = 2, then it will be interpreted as giving standard errors that vary across columns but are constant within any particular column. If S is a matrix or scalar, or if the data matrix is not square, then S_dim should be left unspecified (NULL).

# Value

An initialized flash object (with no factors).

flash\_nullcheck Nullcheck flash factors

# Description

Sets factor/loadings pairs to zero if doing so improves the variational lower bound (ELBO). See flash for examples of usage.

# Usage

```
flash_nullcheck(flash, kset = NULL, remove = TRUE, tol = NULL, verbose = NULL)
```

# Arguments

flash	A flash or flash_fit object.
kset	A vector of integers specifying which factors to nullcheck. If kset = NULL, then all existing factors will be checked.
remove	Whether to remove factors that have been set to zero from the flash object. Note that this might change the indices of existing factors.
tol	The "tolerance" parameter: if a factor does not improve the ELBO by at least tol, then it will be set to zero. Note that flash_nullcheck does not respect "global" tolerance parameters set by flash_set_conv_crit (which only affects the convergence tolerance for greedy fits and backfits). The default tolerance is $np\sqrt{\epsilon}$ , where n is the number of rows in the dataset, p is the number of columns, and $\epsilon$ is equal to .Machine\$double.eps.
verbose	When and how to display progress updates. For nullchecks, updates are only displayed when verbose $> 0$ .

#### Value

The flash object from argument flash, with factors that do not improve the ELBO by at least tol either set to zero or removed (depending on the argument to parameter remove).

#### See Also

flash\_factors\_remove, flash\_factors\_set\_to\_zero

flash\_set\_conv\_crit Set convergence criterion and tolerance parameter

## Description

Used in a flash pipeline to set the criterion for determining whether a greedy fit or backfit has "converged."

#### Usage

flash\_set\_conv\_crit(flash, fn = NULL, tol)

#### Arguments

flash	A flash or flash_fit object.
fn	The convergence criterion function (see Details below). If NULL, then only the tolerance parameter is updated (thus a convergence criterion can be set at the beginning of a flash pipeline, allowing the tolerance parameter to be updated at will without needing to re-specify the convergence criterion each time). The default convergence criterion, which is set when the flash object is initialized, is flash_conv_crit_elbo_diff, which calculates the difference in the variational lower bound or "ELBO" from one iteration to the next.
tol	The tolerance parameter (see Details below). The default, which is set when the flash object is initialized (see flash_init), is $np\sqrt{\epsilon}$ , where n is the number of rows in the dataset, p is the number of columns, and $\epsilon$ is equal to .Machine\$double.eps.

## Details

Function flash\_set\_conv\_crit can be used to customize the convergence criterion for a flash object. This criterion determines when to stop optimizing a newly added factor (see flash\_greedy) and when to stop backfitting (flash\_backfit). Note that, because most alternative convergence criteria do not make sense in the context of a nullcheck, it does not set the "convergence" criterion for flash\_nullcheck (for example, flash\_conv\_crit\_max\_chg\_L would simply return the maximum  $L^2$ -normalized loading for each set of loadings  $\ell_{\cdot k}$ ).

The criterion is defined by the function supplied as argument to fn, which must accept exactly three parameters, curr, prev, and k. curr refers to the flash\_fit object from the current iteration; prev, to the flash\_fit object from the previous iteration; and, if the iteration is a sequential backfitting iteration (that is, a flash\_backfit iteration with argument extrapolate = FALSE), k identifies the

factor/loadings pair that is currently being updated (in all other cases, k is NULL). The function must output a numeric value; if the value is less than or equal to tol, then the fit is considered to have "converged." The meaning of "convergence" here varies according to the operation being performed. In the greedy algorithm, fn simply compares the fit from one iteration to the next. During a backfit, it similarly compares fits from one iteration to the next, but it only considers the fit to have converged when the value of fn over successive updates to *all* factor/loadings pairs is less than or equal to tol. If, for example, factor/loadings pairs  $1, \ldots, K$  are being sequentially backfitted, then fits are compared before and after the update to factor/loadings 1, before and after the update to factor/loadings 2, and so on through factor/loadings *K*, and backfitting only terminates when fn returns a value less than or equal to tol for all *K* updates.

Package flashier provides a number of functions that may be supplied as convergence criteria: see flash\_conv\_crit\_elbo\_diff (the default criterion), flash\_conv\_crit\_max\_chg, flash\_conv\_crit\_max\_chg\_L, and flash\_conv\_crit\_max\_chg\_F. Custom functions may also be defined. Typically, they will compare the fit in curr (the current iteration) to the fit in prev (the previous iteration). To facilitate working with flash\_fit objects, package flashier provides a number of accessors, which are enumerated in the documentation for object flash\_fit. Custom functions should return a numeric value that can be compared against tol; see **Examples** below.

#### Value

The flash object from argument flash, with the new convergence criterion reflected in updates to the "internal" flash\_fit object. These settings will persist across all subsequent calls to flash\_xxx functions in the same flash pipeline (unless, of course, flash\_set\_conv\_crit is again called within the same pipeline).

#### Examples

```
fl <- flash_init(gtex) %>%
flash_set_conv_crit(flash_conv_crit_max_chg, tol = 1e-3) %>%
flash_set_verbose(
  verbose = 3,
  fns = flash_verbose_max_chg,
  colnames = "Max Chg",
  colwidths = 20
) %>%
flash_greedy(Kmax = 3)
```

flash\_set\_timeout Set timeout

## Description

Used in a flash pipeline to set a maximum amount of fitting time. Note that timeout conditions are only checked during greedy fits and backfits, so that the total amount of fitting time can exceed the time set by flash\_set\_timeout (especially if, for example, there is a nullcheck involving many factor/loading pairs). Also note that timeout conditions must be cleared using function flash\_clear\_timeout before any re-fitting is attempted.

30

flash\_set\_verbose

## Usage

```
flash_set_timeout(
   flash,
   tim,
   units = c("hours", "secs", "mins", "days", "weeks")
)
```

# Arguments

flash	A flash or flash_fit object.
tim	A numeric value giving the maximum amount of fitting time, with the units of time specified by parameter units.
units	The units of time according to which parameter tim is to be interpreted.

## Value

The flash object from argument flash, with the timeout settings reflected in updates to the "internal" flash\_fit object. These settings will persist across all subsequent calls to flash\_xxx functions until they are modified either by flash\_clear\_timeout or by another call to flash\_set\_timeout.

# Examples

```
fl <- flash_init(gtex) %>%
  flash_set_timeout(1, "secs") %>%
  flash_greedy(Kmax = 30) %>%
  flash_backfit() %>%
  flash_nullcheck() %>%
  flash_clear_timeout() # Always clear timeout at the end of a pipeline.
```

flash\_set\_verbose Set verbose output

# Description

Used in a flash pipeline to set the output that will be printed after each greedy or backfitting iteration.

## Usage

```
flash_set_verbose(
  flash,
  verbose = 1L,
  fns = NULL,
  colnames = NULL,
  colwidths = NULL
)
```

#### Arguments

flash	A flash or flash_fit object.
verbose	When and how to display progress updates. Set to $\emptyset$ for no updates; 1 for updates after a "greedy" factor is added or a backfit is completed; 2 for additional notifications about the variational lower bound (ELBO); and 3 for updates after every iteration. By default, per-iteration update information includes the change in ELBO and the maximum (absolute) change over all L2-normalized loadings $\ell_1, \ldots, \ell_K$ and factors $f_1, \ldots, f_K$ . Update information is customizable via parameters fns, colnames, and colwidths.
	A single tab-delimited table of values may also be output using option verbose $= -1$ . This format is especially convenient for downstream analysis of the fitting history. For example, it may be used to plot the value of the ELBO after each iteration (see the "Advanced Flashier" vignette for an illustration).
fns	A vector of functions. Used to calculate values to display after each greedy/backfit iteration when verbose is either -1 or 3 (see Details below). Ignored for other values of verbose (0, 1, or 2).
colnames	A vector of column names, one for each function in fns.
colwidths	A vector of column widths, one for each function in fns.

## Details

Function flash\_set\_verbose can be used to customize the output that is printed to console while fitting a flash object. After each greedy or backfitting iteration (see, respectively, flash\_greedy and flash\_backfit), each function in argument fns is successively evaluated and the result is printed to console in a table with column names defined by argument colnames and column widths defined by argument colwidths.

Each function in fns must accept exactly three parameters, curr, prev, and k: curr refers to the flash\_fit object from the current iteration; prev, to the flash\_fit object from the previous iteration; and, if the iteration is a sequential backfitting iteration (that is, a flash\_backfit iteration with argument extrapolate = FALSE), k identifies the factor/loadings pair that is currently being updated (in all other cases, k is NULL). Package flashier provides a number of functions that may be used to customize output: see flash\_verbose\_elbo, flash\_verbose\_elbo\_diff, flash\_verbose\_max\_chg, flash\_verbose\_max\_chg\_L, and flash\_verbose\_max\_chg\_F. Custom functions may also be defined. They might inspect the current flash\_fit object via argument curr; compare the fit in curr to the fit from the previous iteration (provided by argument prev); or ignore both flash\_fit objects entirely (for example, to track progress over time, one might simply call Sys.time). To facilitate working with flash\_fit objects, package flashier provides a number of accessors, which are enumerated in the documentation for object flash\_fit. Custom functions should return a character string that contains the output exactly as it is to displayed; see **Examples** below.

#### Value

The flash object from argument flash, with the new verbose settings reflected in updates to the "internal" flash\_fit object. These settings will persist across all subsequent calls to flash\_xxx functions until they are modified by another call to flash\_set\_verbose.

## Examples

```
# Suppress all verbose output.
fl <- flash_init(gtex) %>%
  flash_set_verbose(0) %>%
  flash_greedy(Kmax = 5)
# Set custom verbose output.
sparsity_F <- function(curr, prev, k) {</pre>
  g_F <- flash_fit_get_g(curr, n = 2)</pre>
  g_F_pi0 <- g_F$pi[1] # Mixture weight of the "null" component.</pre>
  return(g_F_pi0)
}
verbose_fns <- c(flash_verbose_elbo, flash_verbose_max_chg_F, sparsity_F)</pre>
colnames <- c("ELBO", "Max Chg (Tiss)", "Sparsity (Tiss)")</pre>
colwidths <- c(12, 18, 18)
fl <- flash_init(gtex) %>%
  flash_set_verbose(
    verbose = 3,
    fns = verbose_fns,
    colnames = colnames,
    colwidths = colwidths
  ) %>%
  flash_greedy(Kmax = 3)
# Output can be changed as needed.
fl <- flash_init(gtex) %>%
  flash_set_verbose(verbose = 1) %>%
  flash_greedy(Kmax = 5L) %>%
  flash_backfit(verbose = 3) %>%
  flash_greedy(Kmax = 1L)
```

flash\_verbose\_elbo Display the current ELBO

# Description

Displays the value of the variational lower bound (ELBO) at the current iteration.

## Usage

```
flash_verbose_elbo(curr, prev, k)
```

## Arguments

curr	The flash_fit object from the current iteration.
prev	The flash_fit object from the previous iteration.
k	Only used during sequential backfits (that is, calls to flash_backfit where extrapolate = FALSE). It then takes the index of the factor/loadings pair currently being optimized.

## Details

This function is an example of a function that may be passed to parameter fns in function flash\_set\_verbose to customize the output that is printed after each greedy or backfitting iteration. See flash\_set\_verbose for details and examples.

#### Value

A character string, suitable for printing progress updates.

## See Also

flash\_verbose\_elbo\_diff,flash\_verbose\_max\_chg,flash\_verbose\_max\_chg\_L,flash\_verbose\_max\_chg\_F

flash\_verbose\_elbo\_diff

Display the difference in ELBO

#### Description

Displays the difference in the variational lower bound (ELBO) from one iteration to the next.

## Usage

flash\_verbose\_elbo\_diff(curr, prev, k)

# Arguments

curr	The flash_fit object from the current iteration.
prev	The flash_fit object from the previous iteration.
k	Only used during sequential backfits (that is, calls to flash_backfit where extrapolate = FALSE). It then takes the index of the factor/loadings pair currently being optimized.

# Details

This function is an example of a function that may be passed to parameter fns in function flash\_set\_verbose to customize the output that is printed after each greedy or backfitting iteration. See flash\_set\_verbose for details and examples.

## Value

A character string, suitable for printing progress updates.

# See Also

flash\_verbose\_elbo, flash\_verbose\_max\_chg, flash\_verbose\_max\_chg\_L, flash\_verbose\_max\_chg\_F

flash\_verbose\_max\_chg Display the maximum difference in scaled loadings and factors

# Description

Displays the maximum (absolute) change over all (posterior expected values for) loadings  $\ell_{ik}$  and factors  $f_{jk}$ . At each iteration, the loadings vectors  $\ell_{.1}, \ldots, \ell_{.K}$  and factors  $f_{.1}, \ldots, f_{.K}$  are  $L^2$ -normalized.

## Usage

flash\_verbose\_max\_chg(curr, prev, k)

## Arguments

curr	The flash_fit object from the current iteration.
prev	The flash_fit object from the previous iteration.
k	Only used during sequential backfits (that is, calls to flash_backfit where extrapolate = FALSE). It then takes the index of the factor/loadings pair currently being optimized.

## Details

This function is an example of a function that may be passed to parameter fns in function flash\_set\_verbose to customize the output that is printed after each greedy or backfitting iteration. See flash\_set\_verbose for details and examples.

#### Value

A character string, suitable for printing progress updates.

#### See Also

flash\_verbose\_elbo,flash\_verbose\_elbo\_diff,flash\_verbose\_max\_chg\_L,flash\_verbose\_max\_chg\_F

flash\_verbose\_max\_chg\_F

Display the maximum difference in scaled factors

## Description

Displays the maximum (absolute) change over all (posterior expected values for) factors  $f_{jk}$ . At each iteration, the factors  $f_{.1}, \ldots, f_{.K}$  are  $L^2$ -normalized.

## Usage

flash\_verbose\_max\_chg\_F(curr, prev, k)

#### Arguments

curr	The flash_fit object from the current iteration.
prev	The flash_fit object from the previous iteration.
k	Only used during sequential backfits (that is, calls to flash_backfit where extrapolate = FALSE). It then takes the index of the factor/loadings pair currently being optimized.

# Details

This function is an example of a function that may be passed to parameter fns in function flash\_set\_verbose to customize the output that is printed after each greedy or backfitting iteration. See flash\_set\_verbose for details and examples.

# Value

A character string, suitable for printing progress updates.

#### See Also

flash\_verbose\_elbo,flash\_verbose\_elbo\_diff,flash\_verbose\_max\_chg,flash\_verbose\_max\_chg\_L

flash\_verbose\_max\_chg\_L

Display the maximum difference in scaled loadings

#### Description

Displays the maximum (absolute) change over all (posterior expected values for) loadings  $\ell_{ik}$ . At each iteration, the loadings vectors  $\ell_{.1}, \ldots, \ell_{.K}$  are  $L^2$ -normalized.

# Usage

```
flash_verbose_max_chg_L(curr, prev, k)
```

#### Arguments

curr	The flash_fit object from the current iteration.
prev	The flash_fit object from the previous iteration.
k	Only used during sequential backfits (that is, calls to flash_backfit where extrapolate = FALSE). It then takes the index of the factor/loadings pair currently being optimized.

36

#### gtex

#### Details

This function is an example of a function that may be passed to parameter fns in function flash\_set\_verbose to customize the output that is printed after each greedy or backfitting iteration. See flash\_set\_verbose for details and examples.

#### Value

A character string, suitable for printing progress updates.

## See Also

flash\_verbose\_elbo,flash\_verbose\_elbo\_diff,flash\_verbose\_max\_chg,flash\_verbose\_max\_chg\_F

gtex

GTEx data

## Description

Derived from data made available by the Genotype Tissue Expression (GTEx) project (Lonsdale et al. 2013), which provides z-scores for assessing the significance of effects of genetic variants (single nucleotide polymorphisms, or SNPs) on gene expression across 44 human tissues. To reduce the data to a more manageable size, Urbut et al. (2019) chose the "top" SNP for each gene — that is, the SNP associated with the largest (absolute) z-score over all 44 tissues. This yields a  $16,069 \times 44$  matrix of z-scores, with rows corresponding to SNP-gene pairs and columns corresponding to tissues. The dataset included here is further subsampled down to 1000 rows.

#### Format

gtex is a matrix with 1000 rows and 44 columns, with rows corresponding to SNP-gene pairs and columns corresponding to tissues.

#### Source

<https://github.com/stephenslab/gtexresults/blob/master/data/MatrixEQTLSumStats.Portable.Z.rds>

## References

Lonsdale et al. (2013). "The Genotype-Tissue Expression (GTEx) project." *Nature Genetics* 45(6), 580–585.

Urbut, Wang, Carbonetto, and Stephens (2019). "Flexible statistical methods for estimating and testing effects in genomic studies with multiple conditions." *Nature Genetics* 51(1), 187–195.

#### Examples

data(gtex)
summary(gtex)

#### Description

38

A custom palette used by Wang and Stephens (2021) to plot an empirical Bayes matrix factorization of data from the GTEx project (of which the gtex data in package **flashier** is a subsample). The palette is designed to link similar tissues together visually. For example, brain tissues all have the same color (yellow); arterial tissues are shades of pink or red; etc.

#### Format

gtex\_colors is a named vector of length 44, with names corresponding to tissues (columns) in the gtex dataset and values giving hexadecimal color codes.

#### Source

<https://github.com/stephenslab/gtexresults/blob/master/data/GTExColors.txt>

# References

Wei Wang and Matthew Stephens (2021). "Empirical Bayes matrix factorization." *Journal of Machine Learning Research* 22, 1–40.

# Examples

fl <- flash(gtex, greedy\_Kmax = 4)
plot(fl, incl\_scree = FALSE, pm\_colors = gtex\_colors)</pre>

ldf

LDF method for flash and flash fit objects

#### Description

Given a flash or flash\_fit object, returns the LDF decomposition  $Y \approx LDF'$ .

#### Usage

```
ldf(object, type)
## S3 method for class 'flash'
ldf(object, type = "f")
## S3 method for class 'flash_fit'
ldf(object, type = "f")
```

#### plot.flash

#### Arguments

object	An object inheriting from class flash or flash_fit.
type	Takes identical arguments to function norm. Use "f" or "2" for the 2-norm (Euclidean norm); "o" or "1" for the 1-norm (taxicab norm); and "i" or "m" for
	the infinity norm (maximum norm).

## Details

When the prior families  $G_{\ell}^{(k)}$  and  $G_{f}^{(k)}$  are closed under scaling (as is typically the case), then the EBMF model (as described in the documention to function flash) is only identifiable up to scaling by a diagonal matrix D:

$$Y = LDF' + E.$$

Method 1df scales columns  $\ell_k$  and  $f_k$  so that, depending on the argument to parameter type, their 1-norms, 2-norms, or infinity norms are equal to 1.

## Value

A list with fields L, D, and F, each of which corresponds to one of the matrices in the decomposition  $Y \approx LDF'$  (with the columns of L and F scaled according to argument type). Note that D is returned as a vector rather than a matrix (the vector of diagonal entries in D). Thus, "fitted values" LDF' can be recovered as L %\*% diag(D) %\*% t(F).

## Methods (by class)

- ldf(flash): LDF decomposition for flash objects
- ldf(flash\_fit): LDF decomposition for flash\_fit objects

plot.flash

Plot method for flash objects

#### Description

Given a flash object, produces up to two figures: one showing the proportion of variance explained per factor/loadings pair, and one that plots posterior means for either factors or loadings (depending on the argument to parameter pm\_which).

#### Usage

```
## S3 method for class 'flash'
plot(
    x,
    include_scree = TRUE,
    order_by_pve = TRUE,
    kset = NULL,
    pm_which = c("factors", "loadings"),
```

```
pm_subset = NULL,
pm_groups = NULL,
pm_colors = NULL,
....)
```

# Arguments

x	An object inheriting from class flash.
include_scree	Whether to include a figure ("scree plot") showing the proportion of variance explained by each factor/loadings pair.
include_pm	Whether to include a figure showing the posterior means for either loadings $L$ or factors $F$ (depending on the argument to parameter pm_which). One plot panel is produced for each factor/loadings pair $k$ . If argument pm_groups is left unspecified, then bar plots will be produced, with each bar corresponding to a single value $\ell_{ik}$ or $f_{jk}$ . Otherwise, overlapping histograms will be produced, with each histogram corresponding to one of the groups specified by pm_groups.
order_by_pve	If TRUE, then the factor/loadings pairs will be re-ordered according to proportion of variance explained (from highest to lowest).
kset	A vector of integers specifying the factor/loadings pairs to be plotted. If kset = NULL, then all will be plotted.
pm_which	Whether to plot loadings $L$ or factors $F$ in the plots of posterior means. This parameter is ignored when include_pm = FALSE.
pm_subset	A vector of row indices $i$ or column indices $j$ (depending on the argument to pm_which) specifying which values $\ell_i$ . or $f_j$ . are to be shown in the plots of posterior means. If the dataset has row or column names, then names rather than indices may be specified. If pm_subset = NULL, then all values will be plotted. This parameter is ignored when include_pm = FALSE.
pm_groups	A vector specifying the group to which each row of the data $y_i$ . or column $y_{.j}$ belongs (groups may be numeric indices or strings). If pm_groups = NULL, then a bar plot of the ungrouped data is produced (see include_pm above). Otherwise, a group must be provided for each plotted row $i$ or column $j$ , so that the length of pm_groups is exactly equal to the number of rows or columns in the full dataset or, if pm_subset is specified, in the subsetted dataset. When pm_groups is not NULL, a set of overlapping histograms is produced for each factor/loadings pair, with one histogram per group (again see include_pm). This parameter is ignored when include_pm = FALSE.
pm_colors	A vector specifying a color for each bar (if pm_groups = NULL) or histogram (if pm_groups is not NULL). Passed directly to parameter values in <b>ggplot2</b> function scale_color_manual. This parameter is ignored when include_pm = FALSE.
	Additional parameters are ignored.

# Value

If arguments include\_scree and include\_pm specify that only one figure be produced, then plot.flash() returns a ggplot2 object. If both figures are to be produced, then plot.flash()

## residuals.flash

prints both plots but does not return a value.

residuals.flash Residuals method for flash objects

#### Description

Given a flash object, returns the expected residuals Y - E(LF') = Y - E(L)E(F)'.

## Usage

```
## S3 method for class 'flash'
residuals(object, ...)
```

# Arguments

object	An object inheriting from class flash.
	Additional parameters are ignored.

## Value

The matrix of expected residuals.

residuals.flash\_fit Residuals method for flash fit objects

# Description

Given a flash\_fit object, returns the expected residuals Y - E(LF') = Y - E(L)E(F)'.

## Usage

```
## S3 method for class 'flash_fit'
residuals(object, ...)
```

#### Arguments

object	An object inheriting from class flash_fit.
	Additional parameters are ignored.

#### Value

The matrix of expected residuals.

# Index

\* data gtex, 37 gtex\_colors, 38 .Machine, 10, 23, 28, 29 ebnm, 5, 14, 15, 17, 23 fitted.flash, 3, 7 fitted.flash\_fit, 3, 21 flash, 3, 4, 8-20, 22, 24, 27-32, 38, 39, 41 flash\_add\_intercept, 8 flash\_backfit, 7, 8, 9, 11-14, 17, 29, 32-36 flash\_clear\_timeout, 10, 30, 31 flash\_conv\_crit\_elbo\_diff, 11, 12, 13, 29, 30 flash\_conv\_crit\_max\_chg, 11, 12, 13, 30 flash\_conv\_crit\_max\_chg\_F, 11, 12, 12, 13, 30 flash\_conv\_crit\_max\_chg\_L, 11-13, 13, 29, 30 flash\_ebnm, 5, 14, 17, 23 flash\_factors\_fix, 7, 16, 17, 20 flash\_factors\_init, 5, 7, 14-17, 17, 23 flash\_factors\_remove, 7, 18, 19, 29 flash\_factors\_reorder, 19 flash\_factors\_set\_to\_zero, 7, 18, 19, 29 flash\_factors\_unfix, 16, 20 flash\_fit, 3, 7, 11-13, 20, 23, 29, 30, 32-36, 38, 39, 41 flash\_fit\_get\_elbo (flash\_fit), 20 flash\_fit\_get\_est\_tau (flash\_fit), 20 flash\_fit\_get\_fixed\_tau (flash\_fit), 20 flash\_fit\_get\_g (flash\_fit), 20 flash\_fit\_get\_KL (flash\_fit), 20 flash\_fit\_get\_p2m (flash\_fit), 20 flash\_fit\_get\_pm(flash\_fit), 20 flash\_fit\_get\_tau (flash\_fit), 20 flash\_greedy, 5, 7, 14, 15, 17, 20, 22, 23, 25-27, 29, 32

flash\_greedy\_init\_default, 23, 24, 24, 26, 27 flash\_greedy\_init\_irlba, 23-25, 26, 27 flash\_greedy\_init\_softImpute, 23-26, 26 flash\_init, 4, 7, 27, 27, 29 flash\_nullcheck, 7, 28, 29 flash\_set\_conv\_crit, 7, 10, 11, 20, 23, 28, 29 flash\_set\_timeout, 10, 30 flash\_set\_verbose, 5, 7, 10, 20, 24, 31, 34-37 flash\_verbose\_elbo, 32, 33, 34-37 flash\_verbose\_elbo\_diff, 32, 34, 34, 35-37 flash\_verbose\_max\_chg, 32, 34, 35, 36, 37 flash\_verbose\_max\_chg\_F, 32, 34, 35, 35, 37 flash\_verbose\_max\_chg\_L, 32, 34–36, 36 gtex, 37, 38 gtex\_colors, 38 irlba, 4, 26, 27 1df. 38 ldf.flash,7 ldf.flash\_fit, 21 Matrix, 4, 27 norm, 39 plot.flash, 39 residuals.flash, 7, 41 residuals.flash\_fit, 21, 41 rsvd, 4, 27 scale\_color\_manual, 40 softImpute, 4, 26, 27 svd, 4, 27 Sys.time, 32