## Package 'WH'

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Type Package

Title Enhanced Implementation of Whittaker-Henderson Smoothing

Version 2.0.0

Description An enhanced implementation of Whittaker-Henderson smoothing for the graduation of one-dimensional and two-dimensional actuarial tables used to quantify Life Insurance risks.
 'WH' is based on the methods described in Biessy (2025) <doi:10.48550/arXiv.2306.06932>. Among other features, it generalizes the original smoothing algorithm to maximum likelihood estimation,

automatically selects the smoothing parameter(s) and extrapolates beyond the range of data.

License GPL (>= 3)

Encoding UTF-8

LazyData true

URL https://github.com/GuillaumeBiessy/WH

BugReports https://github.com/GuillaumeBiessy/WH/issues

**Depends** R (>= 4.2)

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LinkingTo Rcpp

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**Config/testthat/edition** 3

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WH-package

WH : Enhanced Implementation of Whittaker-Henderson Smoothing

## Description

An enhanced implementation of Whittaker-Henderson smoothing for the gradation of one-dimensional and two-dimensional actuarial tables used to quantify Life Insurance risks. WH is based on the methods described in Biessy (2025) doi:10.48550/arXiv.2306.06932. Among other features, it generalizes the original smoothing algorithm to maximum likelihood estimation, automatically selects the smoothing parameter(s) and extrapolates beyond the range of data.

## Author(s)

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## See Also

Useful links:

- https://github.com/GuillaumeBiessy/WH
- Report bugs at https://github.com/GuillaumeBiessy/WH/issues

output\_to\_df

## Description

Store WH model fit results in a data.frame

## Usage

output\_to\_df(object, dim1 = "x", dim2 = "z")

## Arguments

object	An object of class "WH_1d" or "WH_2d" returned by the WH() function
dim1	The (optional) name to be given to the first dimension
dim2	The (optional) name to be given to the second dimension

## Value

A data.frame gathering information about the fitted and predicted values, the model variance, residuals and effective degrees of freedom...

plot.WH_1d	Plot 1D WH fit	
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## Description

Plot 1D WH fit

## Usage

```
## S3 method for class 'WH_1d'
plot(x, what = "fit", trans, ...)
```

## Arguments

х	An object of class "WH_1d" returned by the WH() function
what	What should be plotted. Should be one of fit (the default), res for residuals and edf for the effective degrees of freedom.
trans	An (optional) transformation to be applied to the data. By default the identity function
	Not used

## Value

A plot representing the desired element of the fit

#### Examples

```
d <- portfolio_mort$d
ec <- portfolio_mort$ec
WH(d, ec) |> plot()
WH(d, ec) |> plot("res")
WH(d, ec) |> plot("edf")
```

plot.WH\_2d

## Plot 2D WH fit

## Description

Plot 2D WH fit

## Usage

```
## S3 method for class 'WH_2d'
plot(x, what = "y_hat", trans, ...)
```

## Arguments

Х	An object of class "WH_2d" returned by the WH() function
what	What should be plotted (y_hat, std_y_hat, res, edf)
trans	An (optional) transformation to be applied to the data
	Not used

## Value

A plot representing the given element of the fit...

```
d <- portfolio_LTC$d
ec <- portfolio_LTC$ec
WH(d, ec) |> plot()
WH(d, ec) |> plot("std_y_hat")
```

portfolio\_LTC

#### Description

Aggregated dataset built from a simulated long-term care portfolio

## Usage

portfolio\_LTC

#### Format

An synthetic aggregated dataset with death and exposure counts from a simulated long-term care portfolio with 100,000 contributors on a 20-year observation period (only deaths following long-term care claims are counted). The dataset is supplied as a list with two components :

- **d** A matrix containing the portfolio number of observed deaths for each combination of age from 70 to 100 (excluded) and duration in LTC from 0 to 15 (excluded)
- ec A matrix containing the portfolio central exposure in person-years for each combination age from 70 to 100 (excluded) and duration in LTC from 0 to 15 (excluded)

portfolio\_mort Aggregated Mortality Dataset

## Description

Aggregated dataset built from a simulated mortality portfolio

#### Usage

portfolio\_mort

## Format

An synthetic aggregated dataset with death and exposure counts from a simulated annuity portfolio with 100,000 contributors on a 20-year observation period. The dataset is supplied as a list with two components :

- **d** A vector containing the portfolio number of observed deaths for each age from 50 to 95 (excluded)
- ec A vector containing the portfolio central exposure in person-years for each age from 50 to 95 (excluded)

predict.WH\_1d

#### Description

Extrapolate the model for new observations.

#### Usage

```
## S3 method for class 'WH_1d'
predict(object, newdata = NULL, ...)
```

#### Arguments

object	An object of class "WH_1d" returned by the WH() function
newdata	A vector containing the position of new observations. Observations from the fit will automatically be added to this, in the adequate order
	Not used

#### Value

An object of class "WH\_1d" with additional components for model prediction.

#### Examples

```
object <- WH(portfolio_mort$d, portfolio_mort$ec)
object_extra <- predict(object, newdata = 40:99)
plot(object_extra)</pre>
```

predict.WH\_2d *Predict new values using a fitted 2D WH model* 

## Description

Extrapolate the model for new observations in a way that is consistent with the fitted values

## Usage

```
## S3 method for class 'WH_2d'
predict(object, newdata = NULL, ...)
```

## Arguments

object	An object of class "WH_2d" returned by the WH() function
newdata	A list containing two vectors indicating the new observation positions
	Not used

## print.WH\_1d

## Value

An object of class "WH\_2d" with additional components for model prediction.

## Examples

```
object <- WH(portfolio_LTC$d, portfolio_LTC$ec)
object_extra <- predict(object, newdata = list(age = 60:109, duration = 0:19))
plot(object_extra)</pre>
```

print.WH\_1d Display of 1D WH object

## Description

Display of 1D WH object

## Usage

```
## S3 method for class 'WH_1d'
print(x, ...)
```

#### Arguments

х	An object of class "WH_1d" returned by the WH() function
	Not used

#### Value

```
Invisibly returns x.
```

```
WH(portfolio_mort$d, portfolio_mort$ec)
```

print.WH\_2d

## Description

Display of 2D WH object

## Usage

```
## S3 method for class 'WH_2d'
print(x, ...)
```

## Arguments

х	An object of class "WH_2d" returned by the WH() function
	Not used

## Value

Invisibly returns x.

## Examples

```
WH(portfolio_LTC$d, portfolio_LTC$ec)
```

vcov.WH\_1d

Compute variance-covariance matrix of fitted 1D WH model

## Description

The variance-covariance matrix may be useful in case confidence intervals are required for quantities derived from the fitted values.

## Usage

## S3 method for class 'WH\_1d'
vcov(object, pred = TRUE, ...)

## Arguments

object	An object of class "WH_1d" returned by the WH() function
pred	Should the variance-covariance matrix include the extrapolated values as well (if any) ?
	Not used

## vcov.WH\_2d

## Value

The variance-covariance matrix for the fitted values

#### Examples

```
object <- WH(portfolio_mort$d, portfolio_mort$ec)
vcov(object)</pre>
```

```
object_extra <- predict(object, newdata = 40:99)
V <- vcov(object_extra)</pre>
```

vcov.WH\_2d

#### Compute variance-covariance matrix of fitted 1D WH model

## Description

The variance-covariance matrix may be useful in case confidence intervals are required for quantities derived from the fitted values.

## Usage

## S3 method for class 'WH\_2d'
vcov(object, pred = TRUE, ...)

#### Arguments

object	An object of class "WH_2d" returned by the WH() function
pred	Should the variance-covariance matrix include the extrapolated values as well (if any) ?
	Not used

#### Value

The variance-covariance matrix for the fitted values

```
object <- WH(portfolio_LTC$d, portfolio_LTC$ec)
V <- vcov(object)
object_extra <- predict(object, newdata = list(age = 60:109, duration = 0:19))
V <- vcov(object_extra)</pre>
```

## Description

Main package function to apply Whittaker-Henderson Smoothing in a survival analysis framework. It takes as input two vectors / matrices of observed events and associated central exposure and estimate a smooth version of the log-hazard rate. Smoothing parameters may be supplied or automatically chosen according to a specific criterion such as "REML" (recommended), "AIC", "BIC" or "GCV". Whittaker-Henderson Smoothing may be applied in a full maximum likelihood framework (strongly recommended) or an asymptotic (approximate) Gaussian framework.

## Usage

WH(d, ec, lambda = NULL, q = 2, criterion, reg, y, wt, verbose = 1, ...)

## Arguments

d	Vector / matrix of observed events whose elements should be named.
ec	Vector / matrix of central exposure. The central exposure corresponds to the sum of the exposure period over the insured population. An individual experiencing an event of interest during the year will no longer be exposed afterwards and the exposure should be reduced accordingly.
lambda	Smoothing parameter. If missing, an optimization procedure will be used to find the optimal smoothing parameter.
q	Order of penalization. Polynoms of degrees q – 1 are considered smooth and therefore unpenalized. The default of 2 should be suitable for most practical applications. Higher orders may cause numerical issues.
criterion	Criterion to be used for the selection of the optimal smoothing parameter. De- fault is "REML" which stands for restricted maximum likelihood. Other options include "AIC", "BIC" and "GCV".
reg	Should an approximate regression framework be used ? framework.
У	Optional vector of observations whose elements should be named. Used only in the regression framework and even in this case will be automatically computed from the d and ec arguments if those are supplied. May be useful when using Whittaker-Henderson smoothing outside of the survival analysis framework.
y wt	Optional vector of observations whose elements should be named. Used only in the regression framework and even in this case will be automatically computed from the d and ec arguments if those are supplied. May be useful when using
	Optional vector of observations whose elements should be named. Used only in the regression framework and even in this case will be automatically computed from the d and ec arguments if those are supplied. May be useful when using Whittaker-Henderson smoothing outside of the survival analysis framework. Optional vector / matrix of weights. As for the observation vector / matrix y, used only in the regression framework and even in this case will be automatically computed if the d argument is supplied. May be useful when using Whittaker-

## WH

## Value

An object of class WH\_1d i.e. a list containing, among other things :

- y The observation vector/matrix, either supplied or computed as y = log(d) log(ec)
- y\_hat The vector/matrix of fitted value
- std\_y\_hat The vector/matrix of standard deviation associated with the fitted value
- · res The vector/matrix of model deviance residuals
- · edf The vector/matrix of effective degrees of freedom associated with each observation
- diagnosis A data.frame with one row containing the effective degrees of freedom of the model, the deviance of the fit as well as the AIC, BIC, GCV and REML criteria

```
d <- portfolio_mort$d
ec <- portfolio_mort$ec

y <- log(d / ec)
y[d == 0 | ec == 0] <- NA
wt <- d

# Maximum likelihood
WH(d, ec) # automatic smoothing parameter selection via REML
WH(d, ec, lambda = 1e2) # fixed smoothing parameter
WH(d, ec, criterion = "GCV") # alternative criterion for smoothing parameter selection
# Regression
WH(y = y, wt = wt) # regression framework is default when y is supplied</pre>
```

```
WH(y = y, wt = wt) # regression framework is default when y is supplied WH(d, ec, reg = TRUE, lambda = 1e2) # forces computation of y from d and ec
```

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