

# Package ‘T2EQ’

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**Type** Package

**Title** Functions for Applying the T<sup>2</sup>-Test for Equivalence

**Version** 1.1

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**Description** Contains functions for applying the T<sup>2</sup>-test for equivalence.

The T<sup>2</sup>-test for equivalence is a multivariate two-sample equivalence test.

Distance measure of the test is the Mahalanobis distance.

For multivariate normally distributed data the T<sup>2</sup>-test for equivalence is exact and UMPI.

The function T2EQ() implements the T<sup>2</sup>-test for equivalence according to Wellek (2010) <DOI:10.1201/ebk1439808184>.

The function T2EQ.dissolution.profiles.hoffelder() implements a variant of the T<sup>2</sup>-test for equivalence according to Hoffelder (2016)

<[http://www.ecv.de/suse\\_item.php?suseId=Z|pi|8430](http://www.ecv.de/suse_item.php?suseId=Z|pi|8430)> for the equivalence comparison of highly variable dissolution profiles.

**License** GPL-3

**NeedsCompilation** no

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## Contents

T2EQ-package	2
ex_data_JoBS	3
ex_data_pharmind	4
T2EQ	5
T2EQ.dissolution.profiles.hoffelder	6

## Index

9

## Description

Contains functions for applying the  $T^2$ -test for equivalence. The  $T^2$ -test for equivalence is a multivariate two-sample equivalence test. Distance measure of the test is the Mahalanobis distance. For multivariate normally distributed data the  $T^2$ -test for equivalence is exact and UMPI. The function T2EQ() implements the  $T^2$ -test for equivalence according to Wellek (2010). The function T2EQ.dissolution.profiles.hoffelder() implements a variant of the  $T^2$ -test for equivalence according to Hoffelder (2016) for the equivalence comparison of highly variable dissolution profiles.

## Details

Index of help topics:

T2EQ	Function for applying the $T^2$ -test for equivalence
T2EQ-package	Functions for Applying the $T^2$ -Test for Equivalence
T2EQ.dissolution.profiles.hoffelder	The $T^2$ -test for equivalence for dissolution data
ex_data_JoBS	Example dataset from Hoffelder et al. (2015)
ex_data_pharmind	Example dataset from Hoffelder (2016)

## Author(s)

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## References

- Wellek, S. (2010), *Testing Statistical Hypotheses of Equivalence and Noninferiority. Second edition.* Boca Raton: Chapman & Hall/CRC.
- Hoffelder, T., Goessl, R., Wellek, S. (2015). Multivariate Equivalence Tests for Use in Pharmaceutical Development. *Journal of Biopharmaceutical Statistics*, 25:3, 417-437. URL: <http://dx.doi.org/10.1080/10543406.2014.920344>
- Hoffelder, T. (2016). Highly Variable Dissolution Profiles: Comparison of  $T^2$ -Test for Equivalence and  $f_2$  Based Methods. *pharmind*, 78:4, 587-592. URL: [http://www.ecv.de/suse\\_item.php?suseId=Z|pi|8430](http://www.ecv.de/suse_item.php?suseId=Z|pi|8430)
- Tsong, Y., Hammerstrom, T., Sathe, P., Shah, V.P. (1996). Statistical Assessment of Mean Differences between two Dissolution Data Sets. *Drug Information Journal*, 30:4, 1105-1112. URL: <http://dx.doi.org/10.1177/009286159603000427>
- EMA (2010). Guidance on the Investigation of Bioequivalence. URL: [http://www.ema.europa.eu/docs/en\\_GB/document\\_library/Scientific\\_guideline/2010/01/WC500070039.pdf](http://www.ema.europa.eu/docs/en_GB/document_library/Scientific_guideline/2010/01/WC500070039.pdf)

## Examples

```
## Not run: A recalculation of the example evaluation in Hoffelder et al. (2015)
can be done with the following code:
## End(Not run)

data(ex_data_JoBS)
REF_JoBS <- cbind(ex_data_JoBS[ which(ex_data_JoBS$Group=='REF'), ]
[c("Diss_15_min","Diss_20_min","Diss_25_min")])
TEST_JoBS <- cbind(ex_data_JoBS[ which(ex_data_JoBS$Group=='TEST'), ]
[c("Diss_15_min","Diss_20_min","Diss_25_min")])
equivalence_margin_JoBS <- 0.74^2
test_T2EQ_JoBS <- T2EQ(X=REF_JoBS,Y=TEST_JoBS,eq_margin = equivalence_margin_JoBS)

## Not run: A recalculation of the results underlying Figure 1 in Hoffelder (2016)
can be done with the following code:
## End(Not run)

data(ex_data_pharmind)
REF_pharmind <- cbind(ex_data_pharmind[ which(ex_data_pharmind$Group=='REF'), ]
[c("Diss_10_min","Diss_20_min","Diss_30_min")])
TEST_pharmind <- cbind(ex_data_pharmind[ which(ex_data_pharmind$Group=='TEST'), ]
[c("Diss_10_min","Diss_20_min","Diss_30_min")])
test_T2EQ.dissolution.profiles.hoffelder_pharmind <-
T2EQ.dissolution.profiles.hoffelder(X=REF_pharmind,Y=TEST_pharmind)
```

ex\_data\_JoBS

Example dataset from Hoffelder et al. (2015)

## Description

Multivariate example dataset of dissolution profiles. Dataset consists of two three-dimensional samples. The names of the three variables are "Diss\_15\_min", "Diss\_20\_min" and "Diss\_25\_min". Variable "Group" discriminates between first sample (Group == "REF") and second sample (Group == "Test"). Sample size is 12 per group.

## Usage

```
data("ex_data_JoBS")
```

## Format

A data frame with 24 observations on the following 4 variables.

Group a factor with levels REF TEST  
Diss\_15\_min a numeric vector  
Diss\_20\_min a numeric vector  
Diss\_25\_min a numeric vector

## Details

Example dataset from Hoffelder et al. (2015).

## Source

Hoffelder, T., Goessl, R., Wellek, S. (2015), "Multivariate Equivalence Tests for Use in Pharmaceutical Development", *Journal of Biopharmaceutical Statistics*, 25:3, 417-437.

## References

URL: <http://dx.doi.org/10.1080/10543406.2014.920344>

## Examples

```
data(ex_data_JoBS)
```

*ex\_data\_pharmind*

*Example dataset from Hoffelder (2016)*

## Description

Multivariate example dataset of dissolution profiles. Dataset consists of two three-dimensional samples. The names of the three variables are "Diss\_10\_min", "Diss\_20\_min" and "Diss\_30\_min". Variable "Group" discriminates between first sample (Group == "REF") and second sample (Group == "Test"). Sample size is 12 per group.

## Usage

```
data("ex_data_pharmind")
```

## Format

A data frame with 24 observations on the following 4 variables.

```
Diss_10_min a numeric vector  
Diss_20_min a numeric vector  
Diss_30_min a numeric vector  
Group a character vector
```

## Details

Example dataset underlying Figure 1 in Hoffelder (2016).

## Source

Hoffelder, T. (2016), "Highly Variable Dissolution Profiles: Comparison of  $T^2$ -Test for Equivalence and  $f_2$  Based Methods", *pharmind*, 78:4, 587-592.

## References

URL: [http://www.ecv.de/suse\\_item.php?suseId=Z|pi|8430](http://www.ecv.de/suse_item.php?suseId=Z|pi|8430)

## Examples

```
data(ex_data_pharmind)
```

T2EQ

*Function for applying the  $T^2$ -test for equivalence*

## Description

The function T2EQ() implements the  $T^2$ -test for equivalence (see Wellek,2010 or Hoffelder et al., 2015). The  $T^2$ -test for equivalence is a multivariate two-sample equivalence test. Distance measure of the test is the Mahalanobis distance.

## Usage

```
T2EQ(X, Y, eq_margin, alpha = 0.05, print.results = TRUE)
```

## Arguments

X	numeric data matrix of the first sample. The rows of X contain the individual observations of the sample, the columns contain the variables/components of the multivariate sample.
Y	numeric data matrix of the second sample. The rows of X contain the individual observations of the sample, the columns contain the variables/components of the multivariate sample.
eq_margin	numeric (>0). The equivalence margin of the test.
alpha	numeric (0<alpha<1). The significance level of the $T^2$ -test for equivalence. Usually set to 0.05 which is the default.
print.results	logical; if TRUE (default) summary statistics and test results are printed in the output. If NO no output is created

## Details

For multivariate normally distributed data the  $T^2$ -test for equivalence is exact and UMPI.

## Value

a data frame; three columns containing the results of the test

p.value	numeric; the p-value of the $T^2$ -test for equivalence
testresult.num	numeric; 0 (null hypothesis of nonequivalence not rejected) or 1 (null hypothesis of nonequivalence rejected, decision in favor of equivalence)
testresult.text	character; test result of the $T^2$ -test for equivalence in text mode

## Author(s)

Thomas Hoffelder <thomas.hoffelder at boehringer-ingelheim.com>

## References

- Wellek, S. (2010), *Testing Statistical Hypotheses of Equivalence and Noninferiority. Second edition.* Boca Raton: Chapman & Hall/CRC.
- Hoffelder, T., Goessl, R., Wellek, S. (2015). Multivariate Equivalence Tests for Use in Pharmaceutical Development. *Journal of Biopharmaceutical Statistics*, 25:3, 417-437. URL: <http://dx.doi.org/10.1080/10543406.2014.920344>

## Examples

```
## Not run: A recalculation of the example evaluation in Hoffelder et al. (2015)
can be done with the following code:
## End(Not run)

data(ex_data_JoBS)
REF_JoBS <- cbind(ex_data_JoBS[ which(ex_data_JoBS$Group=='REF')], ]
[c("Diss_15_min","Diss_20_min","Diss_25_min")])
TEST_JoBS <- cbind(ex_data_JoBS[ which(ex_data_JoBS$Group=='TEST')], ]
[c("Diss_15_min","Diss_20_min","Diss_25_min")])
equivalence_margin_JoBS <- 0.74^2
test_T2EQ_JoBS <- T2EQ(X=REF_JoBS,Y=TEST_JoBS,eq_margin = equivalence_margin_JoBS)
```

T2EQ.dissolution.profiles.hoffelder  
*The  $T^2$ -test for equivalence for dissolution data*

## Description

The function `T2EQ.dissolution.profiles.hoffelder()` implements a variant of the  $T^2$ -test for equivalence analyses of highly variable dissolution profiles (see Hoffelder, 2016). It is a multivariate two-sample equivalence procedure. Distance measure of the test is the Mahalanobis distance.

## Usage

```
T2EQ.dissolution.profiles.hoffelder(X, Y, alpha = 0.05, print.results = TRUE)
```

## Arguments

- |                |   |
|----------------|---|
| <code>X</code> | numeric data matrix of the first sample (REF). The rows of X contain the individual observations of the REF sample, the columns contain the variables/components of the multivariate sample. More precisely, the variables are the measured dissolution time points and the rows contain the individual dissolution profiles. |
|----------------|---|

Y	numeric data matrix of the second sample (TEST). The rows of Y contain the individual observations of the TEST sample, the columns contain the variables/components of the multivariate sample. More precisely, the variables are the measured dissolution time points and the rows contain the individual dissolution profiles.
alpha	numeric ( $0 < \text{alpha} < 1$ ). The significance level of the test. Usually set to 0.05 which is the default.
print.results	logical; if TRUE (default) summary statistics and test results are printed in the output. If NO no output is created

## Details

This function implements a variant of the  $T^2$ -test for equivalence suggested in Hoffelder (2016): The equivalence margin of the test is a compromise between the suggestions of Tsong et al. (1996) and EMA (2010) requirements. See Hoffelder (2016) for a discussion on that equivalence margin.

## Value

a data frame; three columns containing the results of the test

p.value	numeric; the p-value of the equivalence test according to Hoffelder (2016)
testresult.num	numeric; 0 (null hypothesis of nonequivalence not rejected) or 1 (null hypothesis of nonequivalence rejected, decision in favor of equivalence)
testresult.text	character; test result of the test in text mode

## Author(s)

Thomas Hoffelder <thomas.hoffelder at boehringer-ingelheim.com>

## References

- Hoffelder, T. (2016). Highly Variable Dissolution Profiles: Comparison of  $T^2$ -Test for Equivalence and  $f_2$  Based Methods. *pharmind*, 78:4, 587-592. URL: [http://www.ecv.de/suse\\_item.php?suseId=Z|pi|8430](http://www.ecv.de/suse_item.php?suseId=Z|pi|8430)
- Wellek, S. (2010), *Testing Statistical Hypotheses of Equivalence and Noninferiority. Second edition*. Boca Raton: Chapman & Hall/CRC.
- Tsong, Y., Hammerstrom, T., Sathe, P., Shah, V.P. (1996). Statistical Assessment of Mean Differences between two Dissolution Data Sets. *Drug Information Journal*, 30:4, 1105-1112. URL: <http://dx.doi.org/10.1177/00928615960300427>
- EMA (2010). Guidance on the Investigation of Bioequivalence. URL: [http://www.ema.europa.eu/docs/en\\_GB/document\\_library/Scientific\\_guideline/2010/01/WC500070039.pdf](http://www.ema.europa.eu/docs/en_GB/document_library/Scientific_guideline/2010/01/WC500070039.pdf)

## Examples

```
## Not run: A recalculation of the results underlying Figure 1 in Hoffelder (2016)
can be done with the following code:
## End(Not run)
```

```
data(ex_data_pharmind)
REF_pharmind <- cbind(ex_data_pharmind[ which(ex_data_pharmind$Group=='REF')], [
  [c("Diss_10_min","Diss_20_min","Diss_30_min")])
TEST_pharmind <- cbind(ex_data_pharmind[ which(ex_data_pharmind$Group=='TEST')], [
  [c("Diss_10_min","Diss_20_min","Diss_30_min")])
test_T2EQ.dissolutionprofiles.hoffelder_pharmind <-
  T2EQ.dissolutionprofiles.hoffelder(X=REF_pharmind,Y=TEST_pharmind)
```

# Index

- \* **Hotelling's T<sup>2</sup>**
  - T2EQ, [5](#)
  - T2EQ-package, [2](#)
  - T2EQ.dissolution.profiles.hoffelder,
    - [6](#)
- \* **Mahalanobis distance**
  - T2EQ, [5](#)
  - T2EQ-package, [2](#)
  - T2EQ.dissolution.profiles.hoffelder,
    - [6](#)
- \* **datasets**
  - ex\_data\_JoBS, [3](#)
  - ex\_data\_pharmind, [4](#)
- \* **dissolution profiles**
  - T2EQ-package, [2](#)
  - T2EQ.dissolution.profiles.hoffelder,
    - [6](#)
- \* **equivalence**
  - T2EQ, [5](#)
  - T2EQ-package, [2](#)
  - T2EQ.dissolution.profiles.hoffelder,
    - [6](#)
- \* **multivariate normal distribution**
  - T2EQ, [5](#)
  - T2EQ-package, [2](#)
- \* **multivariate statistics**
  - T2EQ, [5](#)
  - T2EQ-package, [2](#)
  - T2EQ.dissolution.profiles.hoffelder,
    - [6](#)
- \* **multivariate**
  - T2EQ, [5](#)
  - T2EQ-package, [2](#)
  - T2EQ.dissolution.profiles.hoffelder,
    - [6](#)
- \* **package**
  - T2EQ-package, [2](#)
- \* **robust**
  - T2EQ, [5](#)