Package 'SMFilter'

January 20, 2025

Title Filtering Algorithms for the State Space Models on the Stiefel Manifold

Version 1.0.3

Description Provides the filtering algorithms for the state space models on the Stiefel manifold as well as the corresponding sampling algorithms for uniform, vector Langevin-Bingham and matrix Langevin-Bingham distributions on the Stiefel manifold.

Depends R (>= 3.0.0)

License GPL-3

Encoding UTF-8

LazyData true

RoxygenNote 6.1.1

URL https://github.com/yukai-yang/SMFilter

BugReports https://github.com/yukai-yang/SMFilter/issues

Suggests knitr, rmarkdown, ggplot2 VignetteBuilder knitr NeedsCompilation no Author Yukai Yang [aut, cre] Maintainer Yukai Yang <yukai.yang@statistik.uu.se> Repository CRAN Date/Publication 2018-12-12 22:20:03 UTC

Contents

Dist2	2
lterModel1	3
lterModel2	4
LB_sm	6
nif_sm	7
lb_sm	8
mModel1	9

FDist2

SimModel2 10 SMFilter 12 version 14

FDist2

Compute the squared Frobenius distance between two matrices.

Description

This function Compute the squared Frobenius distance between two matrices.

Usage

FDist2(mX, mY)

Arguments

mX	a $p \times r$ matrix where $p \ge r$.
mY	another $p \times r$ matrix where $p \ge r$.

Details

The Frobenius distance between two matrices is defined to be

$$d(X,Y) = \sqrt{\operatorname{tr}\{A'A\}}$$

where A = X - Y.

The Frobenius distance is a possible measure of the distance between two points on the Stiefel manifold.

Value

the Frobenius distance.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

```
FDist2(runif_sm(1,4,2)[1,,], runif_sm(1,4,2)[1,,])
```

FilterModel1

Description

This function implements the filtering algorithm for the type one model. See Details part below.

Usage

```
FilterModel1(mY, mX, mZ, beta, mB = NULL, Omega, vD, U0,
  method = "max_1")
```

Arguments

mY	the matrix containing Y_t with dimension $T \times p$.
mX	the matrix containing X_t with dimension $T \times q_1$.
mZ	the matrix containing Z_t with dimension $T \times q_2$.
beta	the β matrix.
mB	the coefficient matrix \boldsymbol{B} before mZ with dimension $p imes q_2$.
Omega	covariance matrix of the errors.
vD	vector of the diagonals of D.
UØ	initial value of the alpha sequence.
method	a string representing the optimization method from c('max_1','max_2','max_3','min_1','min_2').

Details

The type one model on Stiefel manifold takes the form:

$$egin{array}{rcl} m{y}_t &=& m{lpha}_t m{eta}' m{x}_t + m{B} m{z}_t + m{arepsilon}_t \ & \ m{lpha}_{t+1} |m{lpha}_t &\sim& ML(p,r,m{lpha}_t m{D}) \end{array}$$

where y_t is a *p*-vector of the dependent variable, x_t and z_t are explanatory variables wit dimension q_1 and q_2 , x_t and z_t have no overlap, matrix B is the coefficients for z_t , ε_t is the error vector.

The matrices α_t and β have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1 . α_t and β are both non-singular matrices. α_t is time-varying while β is time-invariant.

Furthermore, α_t fulfills the condition $\alpha'_t \alpha_t = I_r$, and therefor it evolves on the Stiefel manifold. $ML(p, r, \alpha_t D)$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\alpha_{t+1}}) = \frac{\operatorname{etr} \left\{ \boldsymbol{D} \boldsymbol{\alpha}_{t}^{\prime} \boldsymbol{\alpha_{t+1}} \right\}}{{}_{0}F_{1}(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^{2})}$$

where etr denotes $\exp(tr())$, and ${}_{0}F_{1}(\frac{p}{2}; \frac{1}{4}D^{2})$ is the (0,1)-type hypergeometric function for matrix.

Value

an array aAlpha containing the modal orientations of alpha in the prediction step.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

```
iT = 50
ip = 2
ir = 1
iqx = 4
iqz=0
ik = 0
Omega = diag(ip)*.1
if(iqx==0) mX=NULL else mX = matrix(rnorm(iT*iqx),iT, iqx)
if(iqz==0) mZ=NULL else mZ = matrix(rnorm(iT*iqz),iT, iqz)
if(ik==0) mY=NULL else mY = matrix(0, ik, ip)
alpha_0 = matrix(c(runif_sm(num=1,ip=ip,ir=ir)), ip, ir)
beta = matrix(c(runif_sm(num=1,ip=ip*ik+iqx,ir=ir)), ip*ik+iqx, ir)
mB=NULL
vD = 100
ret = SimModel1(iT=iT, mX=mX, mZ=mZ, mY=mY, alpha_0=alpha_0, beta=beta, mB=mB, vD=vD, Omega=Omega)
mYY=as.matrix(ret$dData[,1:ip])
fil = FilterModel1(mY=mYY, mX=mX, mZ=mZ, beta=beta, mB=mB, Omega=Omega, vD=vD, U0=alpha_0)
```

FilterModel2

Filtering algorithm for the type two model.

Description

This function implements the filtering algorithm for the type two model. See Details part below.

Usage

```
FilterModel2(mY, mX, mZ, alpha, mB = NULL, Omega, vD, U0,
  method = "max_1")
```

Arguments

mY	the matrix containing Y_t with dimension $T \times p$.
mX	the matrix containing X_t with dimension $T \times q_1$.
mZ	the matrix containing Z_t with dimension $T \times q_2$.

FilterModel2

alpha	the α matrix.
mB	the coefficient matrix \boldsymbol{B} before mZ with dimension $p \times q_2$.
Omega	covariance matrix of the errors.
vD	vector of the diagonals of D.
UØ	initial value of the alpha sequence.
method	a string representing the optimization method from c('max_1','max_2','max_3','min_1','min_2').

Details

The type two model on Stiefel manifold takes the form:

$$egin{array}{rcl} m{y}_t &=& m{lpha}m{eta}_t'm{x}_t + m{B}'m{z}_t + m{arepsilon}_t \ m{eta}_{t+1} |m{eta}_t &\sim& ML(q_1,r,m{eta}_tm{D}) \end{array}$$

where y_t is a *p*-vector of the dependent variable, x_t and z_t are explanatory variables wit dimension q_1 and q_2 , x_t and z_t have no overlap, matrix B is the coefficients for z_t , ε_t is the error vector.

The matrices α and β_t have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1 . α and β_t are both non-singular matrices. β_t is time-varying while α is time-invariant.

Furthermore, β_t fulfills the condition $\beta'_t \beta_t = I_r$, and therefor it evolves on the Stiefel manifold. $ML(p, r, \beta_t D)$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\beta_{t+1}}) = \frac{\operatorname{etr} \left\{ \boldsymbol{D} \boldsymbol{\beta}_t' \boldsymbol{\beta}_{t+1} \right\}}{{}_0 F_1(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^2)}$$

where etr denotes $\exp(tr())$, and ${}_{0}F_{1}(\frac{p}{2}; \frac{1}{4}D^{2})$ is the (0,1)-type hypergeometric function for matrix.

Value

an array aAlpha containing the modal orientations of alpha in the prediction step.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

```
iT = 50
ip = 2
ir = 1
iqx = 4
iqz=0
ik = 0
Omega = diag(ip)*.1
if(iqx==0) mX=NULL else mX = matrix(rnorm(iT*iqx),iT, iqx)
if(iqz==0) mZ=NULL else mZ = matrix(rnorm(iT*iqz),iT, iqz)
```

```
if(ik==0) mY=NULL else mY = matrix(0, ik, ip)
alpha = matrix(c(runif_sm(num=1,ip=ip,ir=ir)), ip, ir)
beta_0 = matrix(c(runif_sm(num=1,ip=ip*ik+iqx,ir=ir)), ip*ik+iqx, ir)
mB=NULL
vD = 100
ret = SimModel2(iT=iT, mX=mX, mZ=mZ, mY=mY, alpha=alpha, beta_0=beta_0, mB=mB, vD=vD)
mYY=as.matrix(ret$dData[,1:ip])
fil = FilterModel2(mY=mYY, mX=mX, mZ=mZ, alpha=alpha, mB=mB, Omega=Omega, vD=vD, U0=beta_0)
```

rmLB_sm

Sample from the matrix Langevin-Bingham on the Stiefel manifold.

Description

This function draws a sample from the matrix Langevin-Bingham on the Stiefel manifold.

Usage

rmLB_sm(num, mJ, mH, mC, mX, ir)

Arguments

num	number of observations or sample size.
mJ	symmetric ip*ip matrix
mH	symmetric ir*ir matrix
mC	ip*ir matrix
mX	ip*ir matrix, the initial value
ir	ir

Details

The matrix Langevin-Bingham distribution on the Stiefel manifold has the density kernel:

$$f(X) \propto \operatorname{etr}\{HX'JX + C'X\}$$

where X satisfies $X'X = I_r$, and H and J are symmetric matrices.

Value

an array containing a sample of draws from the matrix Langevin-Bingham on the Stiefel manifold.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

#' @section References: Hoff, P. D. (2009) "Simulation of the Matrix Bingham—von Mises—Fisher Distribution, With Applications to Multivariate and Relational Data", Journal of Computational and Graphical Statistics, Vol. 18, pp. 438-456.

```
6
```

runif_sm

Description

This function draws a sample from the uniform distribution on the Stiefel manifold.

Usage

runif_sm(num, ip, ir)

Arguments

num	number of observations or sample size.
ip	the first dimension p of the matrix.
ir	the second dimension r of the matrix.

Details

The Stiefel manifold with dimension p and r ($p \ge r$) is a space whose points are r-frames in \mathbb{R}^p . A set of r orthonormal vectors in \mathbb{R}^p is called an r-frame in \mathbb{R}^p . The Stiefel manifold is a collection of $p \times r$ full rank matrices X such that $X'X = I_r$.

Value

an array with dimension num, ip and ir containing a sample of draws from the uniform distribution on the Stiefel manifold.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

runif_sm(10,4,2)

rvlb_sm

Description

This function draws a sample from the vector Langevin-Bingham on the Stiefel manifold.

Usage

rvlb_sm(num, mA, vc, vx)

Arguments

num	number of observations or sample size.
mA	the matrix A which is symmetric ip*ip matrix.
vc	the vector c with dimension ip.
vx	the vector x, the initial value.

Details

The vector Langevin-Bingham distribution on the Stiefel manifold has the density kernel:

 $f(X) \propto \operatorname{etr}\{x'Ax + c'x\}$

where x satisfies x'x = 1, and A is a symmetric matrix.

Value

an array containing a sample of draws from the vector Langevin-Bingham on the Stiefel manifold.

References

Hoff, P. D. (2009) "Simulation of the Matrix Bingham—von Mises—Fisher Distribution, With Applications to Multivariate and Relational Data", Journal of Computational and Graphical Statistics, Vol. 18, pp. 438-456.

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

SimModel1

Description

This function simulates from the type one model on Stiefel manifold. See Details part below.

Usage

```
SimModel1(iT, mX = NULL, mZ = NULL, mY = NULL, alpha_0, beta,
mB = NULL, Omega = NULL, vD, burnin = 100)
```

Arguments

iT	the sample size.
mX	the matrix containing X_t with dimension $T \times q_1$.
mZ	the matrix containing Z_t with dimension $T \times q_2$.
mY	initial values of the dependent variable for $ik-1$ up to 0. If mY = NULL, then no lagged dependent variables in regressors.
alpha_0	the initial alpha, $p \times r$.
beta	the β matrix, iqx+ip*ik, y_1,t-1,y_1,t-2,,y_2,t-1,y_2,t-2,
mB	the coefficient matrix \boldsymbol{B} before mZ with dimension $p \times q_2$.
Omega	covariance matrix of the errors.
vD	vector of the diagonals of D.
burnin	burn-in sample size (matrix Langevin).

Details

The type one model on Stiefel manifold takes the form:

$$egin{array}{rcl} m{y}_t &=& m{lpha}_t m{eta}' m{x}_t + m{B} m{z}_t + m{arepsilon}_t \ & \ m{lpha}_{t+1} |m{lpha}_t &\sim& ML(p,r,m{lpha}_tm{D}) \end{array}$$

where y_t is a *p*-vector of the dependent variable, x_t and z_t are explanatory variables wit dimension q_1 and q_2 , x_t and z_t have no overlap, matrix B is the coefficients for z_t , ε_t is the error vector.

The matrices α_t and β have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1 . α_t and β are both non-singular matrices. α_t is time-varying while β is time-invariant.

Furthermore, α_t fulfills the condition $\alpha'_t \alpha_t = I_r$, and therefor it evolves on the Stiefel manifold. $ML(p, r, \alpha_t D)$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\alpha_{t+1}}) = \frac{\operatorname{etr} \left\{ \boldsymbol{D}\boldsymbol{\alpha}_{t}^{\prime}\boldsymbol{\alpha}_{t+1} \right\}}{{}_{0}F_{1}(\frac{p}{2};\frac{1}{4}\boldsymbol{D}^{2})}$$

where etr denotes $\exp(tr())$, and $_0F_1(\frac{p}{2}; \frac{1}{4}D^2)$ is the (0,1)-type hypergeometric function for matrix. Note that the function does not add intercept automatically.

Value

A list containing the sampled data and the dynamics of alpha.

The object is a list containing the following components:

dData	a data.frame of the sampled data
aAlpha	an array of the $\pmb{\alpha}_t$ with the dimension $T\times p\times r$

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

```
iT = 50 # sample size
ip = 2 # dimension of the dependent variable
ir = 1 # rank number
iqx=2 # number of variables in X
iqz=2 # number of variables in Z
ik = 1 # lag length
if(iqx==0) mX=NULL else mX = matrix(rnorm(iT*iqx),iT, iqx)
if(iqz==0) mZ=NULL else mZ = matrix(rnorm(iT*iqz),iT, iqz)
if(ik==0) mY=NULL else mY = matrix(0, ik, ip)
alpha_0 = matrix(c(runif_sm(num=1,ip=ip,ir=ir)), ip, ir)
beta = matrix(c(runif_sm(num=1,ip=ip*ik+iqx,ir=ir)), ip*ik+iqx, ir)
if(ip*ik+iqz=0) mB=NULL else mB = matrix(c(runif_sm(num=1,ip=(ip*ik+iqz)*ip,ir=1)), ip, ip*ik+iqz)
vD = 50
ret = SimModel1(iT=iT, mX=mX, mZ=mZ, mY=mY, alpha_0=alpha_0, beta=beta, mB=mB, vD=vD)
```

SimModel2

Simulate from the type two state-space Model on Stiefel manifold.

Description

This function simulates from the type two model on Stiefel manifold. See Details part below.

Usage

```
SimModel2(iT, mX = NULL, mZ = NULL, mY = NULL, beta_0, alpha,
mB = NULL, Omega = NULL, vD, burnin = 100)
```

10

SimModel2

Arguments

iT	the sample size.
mX	the matrix containing X_t with dimension $T \times q_1$.
mZ	the matrix containing Z_t with dimension $T \times q_2$.
mY	initial values of the dependent variable for $ik-1$ up to 0. If mY = NULL, then no lagged dependent variables in regressors.
beta_0	the initial beta, iqx+ip*ik, y_1,t-1,y_1,t-2,,y_2,t-1,y_2,t-2,
alpha	the α matrix, $p \times r$.
mB	the coefficient matrix \boldsymbol{B} before mZ with dimension $p imes q_2$.
Omega	covariance matrix of the errors.
vD	vector of the diagonals of D.
burnin	burn-in sample size (matrix Langevin).

Details

The type two model on Stiefel manifold takes the form:

$$egin{array}{rcl} m{y}_t &=& m{lpha}m{eta}_t^\prime m{x}_t + m{B}^\prime m{z}_t + m{arepsilon}_t \ m{eta}_{t+1} |m{eta}_t &\sim& ML(q_1,r,m{eta}_tm{D}) \end{array}$$

where y_t is a *p*-vector of the dependent variable, x_t and z_t are explanatory variables wit dimension q_1 and q_2 , x_t and z_t have no overlap, matrix B is the coefficients for z_t , ε_t is the error vector.

The matrices α and β_t have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1 . α and β_t are both non-singular matrices. β_t is time-varying while α is time-invariant.

Furthermore, β_t fulfills the condition $\beta'_t \beta_t = I_r$, and therefor it evolves on the Stiefel manifold. $ML(p, r, \beta_t D)$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\beta_{t+1}}) = \frac{\operatorname{etr}\left\{\boldsymbol{D}\boldsymbol{\beta}_t'\boldsymbol{\beta_{t+1}}\right\}}{{}_0F_1(\frac{p}{2};\frac{1}{4}\boldsymbol{D}^2)}$$

where etr denotes $\exp(tr())$, and ${}_{0}F_{1}(\frac{p}{2}; \frac{1}{4}D^{2})$ is the (0,1)-type hypergeometric function for matrix. Note that the function does not add intercept automatically.

Value

A list containing the sampled data and the dynamics of beta.

The object is a list containing the following components:

dData	a data.frame of the sampled data
aBeta	an array of the $\boldsymbol{\beta}_t$ with the dimension $T\times q_1\times r$

SMFilter

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Examples

```
iT = 50
ip = 2
ir = 1
iqx =3
iqz=2
ik = 1
if(iqx==0) mX=NULL else mX = matrix(rnorm(iT*iqx),iT, iqx)
if(iqz==0) mZ=NULL else mZ = matrix(rnorm(iT*iqz),iT, iqz)
if(ik==0) mY=NULL else mY = matrix(0, ik, ip)
alpha = matrix(c(runif_sm(num=1,ip=ip,ir=ir)), ip, ir)
beta_0 = matrix(c(runif_sm(num=1,ip=ip*ik+iqx,ir=ir)), ip*ik+iqx, ir)
if(ip*ik+iqz=0) mB=NULL else mB = matrix(c(runif_sm(num=1,ip=(ip*ik+iqz)*ip,ir=1)), ip, ip*ik+iqz)
vD = 50
ret = SimModel2(iT=iT, mX=mX, mZ=mZ, mY=mY, alpha=alpha, beta_0=beta_0, mB=mB, vD=vD)
```

SMFilter

SMFilter: a package implementing the filtering algorithms for the state-space models on the Stiefel manifold.

Description

The package implements the filtering algorithms for the state-space models on the Stiefel manifold. It also implements sampling algorithms for uniform, vector Langevin-Bingham and matrix Langevin-Bingham distributions on the Stiefel manifold.

Details

Two types of the state-space models on the Stiefel manifold are considered.

The type one model on Stiefel manifold takes the form:

where y_t is a *p*-vector of the dependent variable, x_t and z_t are explanatory variables wit dimension q_1 and q_2 , x_t and z_t have no overlap, matrix B is the coefficients for z_t , ε_t is the error vector.

The matrices α_t and β have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1 . α_t and β are both non-singular matrices. α_t is time-varying while β is time-invariant.

12

SMFilter

Furthermore, α_t fulfills the condition $\alpha'_t \alpha_t = I_r$, and therefor it evolves on the Stiefel manifold. $ML(p, r, \alpha_t D)$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\alpha_{t+1}}) = \frac{\operatorname{etr} \left\{ \boldsymbol{D} \boldsymbol{\alpha}_{t}^{\prime} \boldsymbol{\alpha_{t+1}} \right\}}{{}_{0} F_{1}(\frac{p}{2}; \frac{1}{4}\boldsymbol{D}^{2})}$$

where etr denotes $\exp(tr())$, and $_0F_1(\frac{p}{2}; \frac{1}{4}D^2)$ is the (0,1)-type hypergeometric function for matrix. The type two model on Stiefel manifold takes the form:

where y_t is a *p*-vector of the dependent variable, x_t and z_t are explanatory variables wit dimension q_1 and q_2 , x_t and z_t have no overlap, matrix B is the coefficients for z_t , ε_t is the error vector.

The matrices α and β_t have dimensions $p \times r$ and $q_1 \times r$, respectively. Note that r is strictly smaller than both p and q_1 . α and β_t are both non-singular matrices. β_t is time-varying while α is time-invariant.

Furthermore, β_t fulfills the condition $\beta'_t \beta_t = I_r$, and therefor it evolves on the Stiefel manifold. $ML(p, r, \beta_t D)$ denotes the Matrix Langevin distribution or matrix von Mises-Fisher distribution on the Stiefel manifold. Its density function takes the form

$$f(\boldsymbol{\beta_{t+1}}) = \frac{\operatorname{etr}\left\{\boldsymbol{D}\boldsymbol{\beta}_t'\boldsymbol{\beta_{t+1}}\right\}}{{}_0F_1(\frac{p}{2};\frac{1}{4}\boldsymbol{D}^2)}$$

where etr denotes $\exp(tr())$, and ${}_{0}F_{1}(\frac{p}{2}; \frac{1}{4}D^{2})$ is the (0,1)-type hypergeometric function for matrix.

Author and Maintainer

Yukai Yang

Department of Statistics, Uppsala University
<yukai.yang@statistik.uu.se>

References

Yang, Yukai and Bauwens, Luc. (2018) "State-Space Models on the Stiefel Manifold with a New Approach to Nonlinear Filtering", Econometrics, 6(4), 48.

Simulation

SimModel1 simulate from the type one state-space model on the Stiefel manifold.

SimModel2 simulate from the type two state-space model on the Stiefel manifold.

Filtering

FilterModel1 filtering algorithm for the type one model.

FilterModel2 filtering algorithm for the type two model.

Sampling

runif_sm sample from the uniform distribution on the Stiefel manifold.

rvlb_sm sample from the vector Langevin-Bingham distribution on the Stiefel manifold.

rmLB_sm sample from the matrix Langevin-Bingham distribution on the Stiefel manifold.

Other Functions

version shows the version number and some information of the package.

version

Show the version number of some information.

Description

This function shows the version number and some information of the package.

Usage

version()

Author(s)

Yukai Yang, <yukai.yang@statistik.uu.se>

Index

* filtering FilterModel1, 3 FilterModel2,4 * simulation SimModel1, 9 SimModel2, 10 * utils FDist2, 2 rmLB_sm,6 runif_sm,7 rvlb_sm,8 version, 14 FDist2, 2 FilterModel1, 3, 13 FilterModel2, 4, 13 rmLB_sm, 6, 14 runif_sm, 7, 14 rvlb_sm, 8, 14 SimModel1, 9, 13 SimModel2, 10, 13 ${\tt SMFilter}, 12$ SMFilter-package (SMFilter), 12

version, *14*, 14