Package 'SLEMI'

January 20, 2025

Title Statistical Learning Based Estimation of Mutual Information

Version 1.0.2

Description

The implementation of the algorithm for estimation of mutual information and channel capacity from experimental data by classification procedures (logistic regression). Technically, it allows to estimate information-theoretic measures between finite-state input and multivariate, continuous output. Method described in Jetka et al. (2019) <doi:10.1371/journal.pcbi.1007132>.

Depends R (>= 3.6.0)

License GPL (>= 3)

URL https://github.com/TJetka/SLEMI

BugReports https://github.com/TJetka/SLEMI/issues

Encoding UTF-8

LazyData true

Imports e1071, ggplot2, gridExtra, nnet, Hmisc, reshape2, stringr, doParallel, caret, corrplot, foreach, methods

Suggests knitr, rmarkdown, testthat (>= 2.1.0), data.table, covr

VignetteBuilder knitr

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NeedsCompilation no

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capacity_logreg_algorithm

Main algorithm to calculate channel capacity by SLEMI approach

Description

Additional parameters: lr_maxit and maxNWts are the same as in definition of multinom function from nnet package. An alternative model formula (using formula_string arguments) should be provided if data are not suitable for description by logistic regression (recommended only for advanced users). It is recommended to conduct estimation by calling capacity_logreg_main.R.

Usage

```
capacity_logreg_algorithm(
  data,
  signal = "signal",
  response = "response",
  side_variables = NULL,
  formula_string = NULL,
  model_out = TRUE,
  cc_maxit = 100,
  lr_maxit = 1000,
  MaxNWts = 5000
)
```

Arguments

data	must be a data.frame object. Cannot contain NA values.
signal	is a character object with names of columns of dataRaw to be treated as chan- nel's input.
response	is a character vector with names of columns of dataRaw to be treated as chan- nel's output
side_variables	(optional) is a character vector that indicates side variables' columns of data, if NULL no side variables are included
formula_string	(optional) is a character object that includes a formula syntax to use in logistic regression model. If NULL, a standard additive model of response variables is assumed. Only for advanced users.

model_out	is the logical indicating if the calculated logistic regression model should be included in output list
cc_maxit	is the number of iteration of iterative optimisation of the algorithm to estimate channel capacity. Default is 100.
lr_maxit	is a maximum number of iteration of fitting algorithm of logistic regression. Default is 1000.
MaxNWts	is a maximum acceptable number of weights in logistic regression algorithm. Default is 5000.

Value

a list with three elements:

- output\$cc channel capacity in bits
- output\$p_opt optimal probability distribution
- · output\$regression confusion matrix of logistic regression predictions
- output\$model nnet object describing logistic regression model (if model_out=TRUE)

References

[1] Jetka T, Nienaltowski K, Winarski T, Blonski S, Komorowski M, Information-theoretic analysis of multivariate single-cell signaling responses using SLEMI, *PLoS Comput Biol*, 15(7): e1007132, 2019, https://doi.org/10.1371/journal.pcbi.1007132.

Examples

```
tempdata=data_example1
outputCLR1=capacity_logreg_algorithm(data=tempdata, signal="signal",
response="response",cc_maxit=3,model_out=FALSE,
formula_string = "signal~response")
```

capacity_logreg_main Estimate channel capacity between discrete input and continuous output

Description

The main wrapping function for basic usage of SLEMI package for estimation of channel capacity. Firstly, data is pre-processed (all arguments are checked, observation with NAs are removed, variables are scaled and centered (if scale=TRUE)). Then basic estimation is carried out and (if testing=TRUE) diagnostic tests are computed. If output directory path is given (output_path is not NULL), graphs visualising the data and the analysis are saved there, together with a compressed output object (as .rds file) with full estimation results.

Usage

```
capacity_logreg_main(
  dataRaw,
  signal = "input",
  response = NULL,
  output_path = NULL,
  side_variables = NULL,
  formula_string = NULL,
  cc_maxit = 100,
  lr_maxit = 1000,
 MaxNWts = 5000,
  testing = FALSE,
 model_out = TRUE,
  scale = TRUE,
  TestingSeed = 1234,
  testing_cores = 1,
  boot_num = 10,
  boot_prob = 0.8,
  sidevar_num = 10,
  traintest_num = 10,
 partition_trainfrac = 0.6,
 plot_width = 6,
 plot_height = 4,
 data_out = FALSE
)
```

Arguments

dataRaw	must be a data.frame object
signal	is a character object with names of columns of dataRaw to be treated as chan- nel's input.
response	is a character vector with names of columns of dataRaw to be treated as channel's output
output_path	is the directory in which output will be saved
side_variables	(optional) is a character vector that indicates side variables' columns of data, if NULL no side variables are included
formula_string	(optional) is a character object that includes a formula syntax to use in logistic regression model. If NULL, a standard additive model of response variables is assumed. Only for advanced users.
cc_maxit	is the number of iteration of iterative optimisation of the algorithm to estimate channel capacity. Default is 100.
lr_maxit	is a maximum number of iteration of fitting algorithm of logistic regression. Default is 1000.
MaxNWts	is a maximum acceptable number of weights in logistic regression algorithm. Default is 5000.
testing	is the logical indicating if the testing procedures should be executed

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model_out	is the logical indicating if the calculated logistic regression model should be included in output list
scale	is a logical indicating if the response variables should be scaled and centered before fitting logistic regression
TestingSeed	is the seed for random number generator used in testing procedures
<pre>testing_cores</pre>	- number of cores to be used in parallel computing (via doParallel package)
boot_num	is the number of bootstrap tests to be performed. Default is 10, but it is recom- mended to use at least 50 for reliable estimates.
boot_prob	is the proportion of initial size of data to be used in bootstrap
sidevar_num	is the number of re-shuffling tests of side variables to be performed. Default is 10, but it is recommended to use at least 50 for reliable estimates.
traintest_num	is the number of overfitting tests to be performed. Default is 10, but it is recom- mended to use at least 50 for reliable estimates.
partition_trainfrac	
	is the fraction of data to be used as a training dataset
plot_width	- basic dimensions (width) of plots, in inches
plot_height	- basic dimensions (height) of plots, in inches
data_out	is the logical indicating if the data should be included in output list

Details

In a typical experiment aimed to quantify information flow a given signaling system, input values $x_1 \leq x_2 \dots \leq x_m$, ranging from 0 to saturation are considered. Then, for each input level, x_i , n_i observations are collected, which are represented as vectors

$$y_i^i \sim P(Y|X=x_i)$$

Within information theory the degree of information transmission is measured as the mutual information

$$MI(X,Y) = \sum_{i=1}^{m} P(x_i) \int_{R^k} P(y|X=x_i) \log_2 \frac{P(y|X=x_i)}{P(y)} dy,$$

where P(y) is the marginal distribution of the output. MI is expressed in bits and 2^{MI} can be interpreted as the number of inputs that the system can resolve on average.

The maximization of mutual information with respect to the input distribution, P(X), defines the information capacity, C. Formally,

$$C^* = max_{P(X)}MI(X,Y)$$

Information capacity is expressed in bits and 2^{C^*} can be interpreted as the maximal number of inputs that the system can effectively resolve.

In contrast to existing approaches, instead of estimating, possibly highly dimensional, conditional output distributions $P(Y|X = x_i)$, we propose to estimate the discrete, conditional input distribution, $P(x_i|Y = y)$, which is known to be a simpler problem. Estimation of the MI using estimates of $P(x_i|Y = y)$, denoted here as $\hat{P}(x_i|Y = y)$, is possible as the MI, can be alternatively written as

$$MI(X,Y) = \sum_{i=1}^{m} P(x_i) \int_{R^k} P(y|X=x_i) \log_2 \frac{P(x_i|Y=y)}{P(x_i)} dy$$

The expected value (as in above expression) with respect to distribution $P(Y|X = x_i)$ can be approximated by the average with respect to data

$$MI(X,Y) \approx \sum_{i=1}^{m} P(x_i) \frac{1}{n_i} \sum_{j=1}^{n_i} P(y|X=x_i) \log_2 \frac{\hat{P}(x_i|Y=y_j^i)}{P(x_i)} dy$$

Here, we propose to use logistic regression as $\hat{P}(x_i|Y = y_i^i)$. Specifically,

$$log \frac{P(x_i|Y=y)}{P(x_m|Y=y)} \approx \alpha_i + \beta_i y$$

Following this approach, channel capacity can be calculated by optimising MI with respect to the input distribution, P(X). However, this, potentially difficult problem, can be divided into two simpler maximization problems, for which explicit solutions exist. Therefore, channel capacity can be obtained from the two explicit solutions in an iterative procedure known as alternate maximization (similarly as in Blahut-Arimoto algorithm) [1].

Additional parameters: lr_maxit and maxNWts are the same as in definition of multinom function from nnet package. An alternative model formula (using formula_string arguments) should be provided if data are not suitable for description by logistic regression (recommended only for advanced users). Preliminary scaling of data (argument scale) should be used similarly as in other data-driven approaches, e.g. if response variables are comparable, scaling (scale=FALSE) can be omitted, while if they represent different phenomenon (varying by units and/or magnitude) scaling is recommended.

Value

a list with several elements:

- · output\$regression confusion matrix of logistic regression predictions
- output\$cc channel capacity in bits
- output\$p_opt optimal probability distribution
- output\$model nnet object describing logistic regression model (if model_out=TRUE)
- output\$params parameters used in algorithm
- · output\$time computation time of calculations
- output\$testing a 2- or 4-element output list of testing procedures (if testing=TRUE)
- output\$testing_pv one-sided p-values of testing procedures (if testing=TRUE)
- output\$data raw data used in analysis

References

[1] Csiszar I, Tusnady G, Information geometry and alternating minimization procedures, Statistics & Decisions 1 Supplement 1 (1984), 205–237.

[2] Jetka T, Nienaltowski K, Winarski T, Blonski S, Komorowski M, Information-theoretic analysis of multivariate single-cell signaling responses using SLEMI, *PLoS Comput Biol*, 15(7): e1007132, 2019, https://doi.org/10.1371/journal.pcbi.1007132.

data_nfkb

Examples

```
tempdata=data_example1
outputCLR1=capacity_logreg_main(dataRaw=tempdata,
signal="signal", response="response",cc_maxit = 10,
formula_string = "signal~response")
tempdata=data_example2
outputCLR2=capacity_logreg_main(dataRaw=tempdata,
signal="signal", response=c("X1","X2"),cc_maxit = 10,
formula_string = "signal~X1+X2")
```

#For further details see vignette

data_nfkb

Data from experiment with NFkB pathway

Description

In the paper describing methodological aspects of our algorithm we present the analysis of information transmission in NfkB pathway upon the stimulation of TNF- α . Experimental data from this experiment in the form of single-cell time series are attached to the package as a data.frame object and can be accessed using 'data_nfkb' variable. Each row of 'data_nfkb' represents a single observation of a cell. Column 'signal' indicates the level of TNF- α stimulation for a given cell, while columns 'response_T', gives the normalised ratio of nuclear and cytoplasmic transcription factor as described in Supplementary Methods of the corresponding publication. In the CRAN version of the package we included only a subset of the data (5 time measurements). For the full datasets, please access GitHub pages.

Usage

data_nfkb

Format

A data frame with 15632 rows and 6 variables:

signal Level of TNFa stimulation

response_0 The concentration of normalised NfkB transcription factor, measured at time 0

response_3 The concentration of normalised NfkB transcription factor, measured at time 3

response_21 The concentration of normalised NfkB transcription factor, measured at time 21

response_90 The concentration of normalised NfkB transcription factor, measured at time 90

response_120 The concentration of normalised NfkB transcription factor, measured at time 120 #'

Details

For each concentration, there are at least 1000 single-cell observation (with the exception of 0.5ng stimulation, where the number of identified cells is almost 900)

Source

in-house experimental data

mi_logreg_algorithm Main algorithm to calculate mutual information by SLEMI approach

Description

Additional parameters: lr_maxit and maxNWts are the same as in definition of multinom function from nnet package. An alternative model formula (using formula_string arguments) should be provided if data are not suitable for description by logistic regression (recommended only for advanced users). It is recommended to conduct estimation by calling mi_logreg_main.R.

Usage

```
mi_logreg_algorithm(
    data,
    signal = "signal",
    response = "response",
    side_variables = NULL,
    pinput = NULL,
    formula_string = NULL,
    lr_maxit = 1000,
    MaxNWts = 5000,
    model_out = TRUE
)
```

Arguments

data	must be a data.frame object. Cannot contain NA values.
signal	is a character object with names of columns of dataRaw to be treated as chan- nel's input.
response	is a character vector with names of columns of dataRaw to be treated as channel's output
side_variables	(optional) is a character vector that indicates side variables' columns of data, if NULL no side variables are included
pinput	is a numeric vector with prior probabilities of the input values. Uniform distribution is assumed as default (pinput=NULL).
formula_string	(optional) is a character object that includes a formula syntax to use in logistic regression model. If NULL, a standard additive model of response variables is assumed. Only for advanced users.
lr_maxit	is a maximum number of iteration of fitting algorithm of logistic regression. Default is 1000.
MaxNWts	is a maximum acceptable number of weights in logistic regression algorithm. Default is 5000.
model_out	is the logical indicating if the calculated logistic regression model should be included in output list

mi_logreg_main

Value

a list with three elements:

- output\$mi mutual information in bits
- output\$pinput prior probabilities used in estimation
- output\$regression confusion matrix of logistic regression model
- output\$model nnet object describing logistic regression model (if model_out=TRUE)

References

[1] Jetka T, Nienaltowski K, Winarski T, Blonski S, Komorowski M, Information-theoretic analysis of multivariate single-cell signaling responses using SLEMI, *PLoS Comput Biol*, 15(7): e1007132, 2019, https://doi.org/10.1371/journal.pcbi.1007132.

Examples

```
mi_logreg_main
```

Estimate mutual information between discrete input and continuous output

Description

The main wrapping function for basic usage of SLEMI package for estimation of mutual information. Firstly, data is pre-processed (all arguments are checked, observation with NAs are removed, variables are scaled and centered (if scale=TRUE)). Then basic estimation is carried out and (if testing=TRUE) diagnostic tests are computed. If output directory path is given (output_path is not NULL), graphs visualising the data and the analysis are saved there, together with a compressed output object (as .rds file) with full estimation results.

Usage

```
mi_logreg_main(
   dataRaw,
   signal = "input",
   response = NULL,
   output_path = NULL,
   side_variables = NULL,
   pinput = NULL,
   formula_string = NULL,
   lr_maxit = 1000,
```

```
MaxNWts = 5000,
testing = FALSE,
model_out = TRUE,
scale = TRUE,
TestingSeed = 1234,
testing_cores = 1,
boot_num = 10,
boot_prob = 0.8,
sidevar_num = 10,
traintest_num = 10,
partition_trainfrac = 0.6,
plot_width = 6,
plot_height = 4,
data_out = FALSE
)
```

Arguments

dataRaw	must be a data.frame object
signal	is a character object with names of columns of dataRaw to be treated as chan- nel's input.
response	is a character vector with names of columns of dataRaw to be treated as chan- nel's output
output_path	is the directory in which output will be saved
side_variables	(optional) is a character vector that indicates side variables' columns of data, if NULL no side variables are included
pinput	is a numeric vector with prior probabilities of the input values. Uniform distribution is assumed as default (pinput=NULL).
formula_string	(optional) is a character object that includes a formula syntax to use in logistic regression model. If NULL, a standard additive model of response variables is assumed. Only for advanced users.
lr_maxit	is a maximum number of iteration of fitting algorithm of logistic regression. Default is 1000.
MaxNWts	is a maximum acceptable number of weights in logistic regression algorithm. Default is 5000.
testing	is the logical indicating if the testing procedures should be executed
model_out	is the logical indicating if the calculated logistic regression model should be included in output list
scale	is a logical indicating if the response variables should be scaled and centered before fitting logistic regression
TestingSeed	is the seed for random number generator used in testing procedures
testing_cores	- number of cores to be used in parallel computing (via doParallel package)
boot_num	is the number of bootstrap tests to be performed. Default is 10, but it is recommended to use at least 50 for reliable estimates.

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boot_prob	is the proportion of initial size of data to be used in bootstrap
sidevar_num	is the number of re-shuffling tests of side variables to be performed. Default is 10, but it is recommended to use at least 50 for reliable estimates.
traintest_num	is the number of overfitting tests to be performed. Default is 10, but it is recom- mended to use at least 50 for reliable estimates.
partition_train	nfrac
	is the fraction of data to be used as a training dataset
plot_width	- basic dimensions (width) of plots, in inches
plot_height	- basic dimensions (height) of plots, in inches
data_out	is the logical indicating if the data should be included in output list

Details

In a typical experiment aimed to quantify information flow a given signaling system, input values $x_1 \leq x_2 \dots \leq x_m$, ranging from 0 to saturation are considered. Then, for each input level, x_i , n_i observations are collected, which are represented as vectors

$$y_i^i \sim P(Y|X=x_i)$$

Within information theory the degree of information transmission is measured as the mutual information

$$MI(X,Y) = \sum_{i=1}^{m} P(x_i) \int_{R^k} P(y|X=x_i) \log_2 \frac{P(y|X=x_i)}{P(y)} dy,$$

where P(y) is the marginal distribution of the output. MI is expressed in bits and 2^{MI} can be interpreted as the number of inputs that the system can resolve on average.

In contrast to existing approaches, instead of estimating, possibly highly dimensional, conditional output distributions $P(Y|X = x_i)$, we propose to estimate the discrete, conditional input distribution, $P(x_i|Y = y)$, which is known to be a simpler problem. Estimation of the MI using estimates of $P(x_i|Y = y)$, denoted here as $\hat{P}(x_i|Y = y)$, is possible as the MI, can be alternatively written as

$$MI(X,Y) = \sum_{i=1}^{m} P(x_i) \int_{R^k} P(y|X=x_i) \log_2 \frac{P(x_i|Y=y)}{P(x_i)} dy$$

The expected value (as in above expression) with respect to distribution $P(Y|X = x_i)$ can be approximated by the average with respect to data

$$MI(X,Y) \approx \sum_{i=1}^{m} P(x_i) \frac{1}{n_i} \sum_{j=1}^{n_i} P(y|X=x_i) \log_2 \frac{\hat{P}(x_i|Y=y_j^i)}{P(x_i)} dy$$

Here, we propose to use logistic regression as $\hat{P}(x_i|Y = y_i^i)$. Specifically,

$$log \frac{P(x_i|Y=y)}{P(x_m|Y=y)} \approx \alpha_i + \beta_i y$$

Additional parameters: lr_maxit and maxNWts are the same as in definition of multinom function from nnet package. An alternative model formula (using formula_string arguments) should be provided if data are not suitable for description by logistic regression (recommended only for advanced users). Preliminary scaling of data (argument scale) should be used similarly as in other data-driven approaches, e.g. if response variables are comparable, scaling (scale=FALSE) can be omitted, while if they represent different phenomenon (varying by units and/or magnitude) scaling is recommended.

Value

a list with several elements:

- · output\$regression confusion matrix of logistic regression predictions
- output\$mi mutual information in bits
- output\$model nnet object describing logistic regression model (if model_out=TRUE)
- output\$params parameters used in algorithm
- output\$time computation time of calculations
- output\$testing a 2- or 4-element output list of testing procedures (if testing=TRUE)
- output\$testing_pv one-sided p-values of testing procedures (if testing=TRUE)
- output\$data raw data used in analysis

References

[1] Jetka T, Nienaltowski K, Winarski T, Blonski S, Komorowski M, Information-theoretic analysis of multivariate single-cell signaling responses using SLEMI, *PLoS Comput Biol*, 15(7): e1007132, 2019, https://doi.org/10.1371/journal.pcbi.1007132.

Examples

```
tempdata=data_example1
outputCLR1=mi_logreg_main(dataRaw=tempdata, signal="signal", response="response")
tempdata=data_example2
outputCLR2=mi_logreg_main(dataRaw=tempdata, signal="signal", response=c("X1","X2","X3"))
```

#For further details see vignette

prob_discr_pairwise Calculates Probability of pairwise discrimination

Description

Estimates probabilities of correct discrimination (PCDs) between each pair of input/signal values using a logistic regression model.

prob_discr_pairwise

Usage

```
prob_discr_pairwise(
    dataRaw,
    signal = "input",
    response = NULL,
    side_variables = NULL,
    formula_string = NULL,
    output_path = NULL,
    scale = TRUE,
    lr_maxit = 1000,
    MaxNWts = 5000,
    diagnostics = TRUE
)
```

Arguments

dataRaw	must be a data.frame object
signal	is a character object with names of columns of dataRaw to be treated as chan- nel's input.
response	is a character vector with names of columns of dataRaw to be treated as chan- nel's output
side_variables	(optional) is a character vector that indicates side variables' columns of data, if NULL no side variables are included
formula_string	(optional) is a character object that includes a formula syntax to use in logistic regression model. If NULL, a standard additive model of response variables is assumed. Only for advanced users.
output_path	is a directory where a pie chart with calculated probabilities will be saved. If NULL, the graph will not be created.
scale	is a logical indicating if the response variables should be scaled and centered before fitting logistic regression
lr_maxit	is a maximum number of iteration of fitting algorithm of logistic regression. Default is 1000.
MaxNWts	is a maximum acceptable number of weights in logistic regression algorithm. Default is 5000.
diagnostics	is a logical indicating if details of logistic regression fitting should be included in output list

Details

In order to estimate PCDs, for a given pair of input values x_i and x_j , we propose to fit a logistic regression model using response data corresponding to the two considered inputs, i.e. y_u^l , for $l \in \{i, j\}$ and u ranging from 1 to n_l . To ensure that both inputs have equal contribution to the calculated discriminability, equal probabilities should be assigned, $P(X) = (P(x_i), P(x_j)) = (1/2, 1/2)$. Once the regression model is fitted, probability of assigning a given cellular response, y, to the correct input value is estimated as

$$\max\{\hat{P}_{lr}(x_i|Y=y; P(X)), \hat{P}_{lr}(x_j|Y=y; P(X))\}.$$

Note that $P(x_j|Y = y) = 1 - P(x_i|Y = y)$ as well as $\hat{P}_{lr}(x_j|Y = y; P(X)) = 1 - \hat{P}_{lr}(x_i|Y = y; P(X))$ The average of the above probabilities over all observations y_l^i yields PCDs

$$\begin{split} PCD_{x_i,x_j} &= \frac{1}{2} \frac{1}{n_i} \sum_{l=1}^{n_i} \max\{\hat{P}_{lr}(x_i | Y = y_i^l; P(X)), \hat{P}_{lr}(x_i^l | Y = y; P(X))\} + \\ &\frac{1}{2} \frac{1}{n_j} \sum_{l=1}^{n_j} \max\{\hat{P}_{lr}(x_i | Y = y_j^l; P(X)), \hat{P}_{lr}(x_j | Y = y_j^l; P(X))\}. \end{split}$$

Additional parameters: lr_maxit and maxNWts are the same as in definition of multinom function from nnet package. An alternative model formula (using formula_string arguments) should be provided if data are not suitable for description by logistic regression (recommended only for advanced users). Preliminary scaling of data (argument scale) should be used similarly as in other data-driven approaches, e.g. if response variables are comparable, scaling (scale=FALSE) can be omitted, while if they represent different phenomenon (varying by units and/or magnitude) scaling is recommended.

Value

a list with two elements:

- output\$prob_matr a $n \times n$ matrix, where n is the number of inputs, with probabilities of correct discrimination between pairs of input values.
- output\$diagnostics (if diagnostics=TRUE) a list corresponding to logistic regression models fitted for each pair of input values. Each element consists of three sub-elements: 1) nnet_model nnet object summarising logistic regression model; 2) prob_lr probabilities of assignment obtained from logistic regression model; 3) confusion_matrix confusion matrix of classificator.

References

[1] Jetka T, Nienaltowski K, Winarski T, Blonski S, Komorowski M, Information-theoretic analysis of multivariate single-cell signaling responses using SLEMI, *PLoS Comput Biol*, 15(7): e1007132, 2019, https://doi.org/10.1371/journal.pcbi.1007132.

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