Package 'MRCE'

January 20, 2025

Type Package

Title Multivariate Regression with Covariance Estimation

Version 2.4

Date 2022-01-04

Author Adam J. Rothman

Maintainer Adam J. Rothman <arothman@umn.edu>

Depends R (>= 2.10.1), glasso

Description Compute and select tuning parameters for the MRCE estimator proposed by Rothman, Levina, and Zhu (2010) <doi:10.1198/jcgs.2010.09188>. This estimator fits the multiple output linear regression model with a sparse estimator of the error precision matrix and a sparse estimator of the regression coefficient matrix.

NeedsCompilation yes

Repository CRAN

Date/Publication 2022-01-04 17:30:10 UTC

Contents

MRCE-package *Multivariate regression with covariance estimation*

Description

Computes the MRCE estimators (Rothman, Levina, and Zhu, 2010) and has the dataset stock04 used in Rothman, Levina, and Zhu (2010), originally analyzed in Yuan et al. (2007).

Details

The primary function is `mrce`. The dataset is `stock04`.

Author(s)

Adam J. Rothman

Maintainer: Adam J. Rothman <arothman@umn.edu>

References

Rothman, A.J., Levina, E., and Zhu, J. (2010). Sparse multivariate regression with covariance estimation. Journal of Computational and Graphical Statistics 19:974–962.

Yuan, M., Ekici, A., Lu, Z., and Monteiro, R. (2007). Dimension reduction and coefficient estimation in multivariate linear regression. Journal of the Royal Statistical Society Series B 69(3):329–346.

Jerome Friedman, Trevor Hastie, Robert Tibshirani (2008). Sparse inverse covariance estimation with the graphical lasso. Biostatistics, 9(3), 432-441.

Jerome Friedman, Trevor Hastie, Robert Tibshirani (2010). Regularization Paths for Generalized Linear Models via Coordinate Descent. Journal of Statistical Software, 33(1), 1-22.

`mrce` *Do multivariate regression with covariance estimation (MRCE)*

Description

Let S_+^q be the set of q by q symmetric and positive definite matrices and let $y_i \in R^q$ be the measurements of the q responses for the ith subject ($i = 1, \dots, n$). The model assumes that y_i is a realization of the q-variate random vector

$$Y_i = \mu + \beta' x_i + \varepsilon_i, \quad i = 1, \dots, n$$

where $\mu \in R^q$ is an unknown intercept vector; $\beta \in R^{p \times q}$ is an unknown regression coefficient matrix; $x_i \in R^p$ is the known vector of values for ith subjects's predictors, and $\varepsilon_1, \dots, \varepsilon_n$ are n independent copies of a q-variate Normal random vector with mean 0 and unknown inverse covariance matrix $\Omega \in S_+^q$.

This function computes penalized likelihood estimates of the unknown parameters μ, β, and Ω. Let $\bar{y} = n^{-1} \sum_{i=1}^n y_i$ and $\bar{x} = n^{-1} \sum_{i=1}^n x_i$. These estimates are

$$(\hat{\beta}, \hat{\Omega}) = \arg \min_{(B,Q) \in R^{p \times q} \times S_+^q} \left\{ g(B,Q) + \lambda_1 \left(\sum_{j \neq k} |Q_{jk}| + 1(p \geq n) \sum_{j=1}^q |Q_{jj}| \right) + 2\lambda_2 \sum_{j=1}^p \sum_{k=1}^q |B_{jk}| \right\}$$

and $\hat{\mu} = \bar{y} - \hat{\beta}' \bar{x}$, where

$$g(B,Q) = \mathrm{tr}\{n^{-1}(Y - XB)'(Y - XB)Q\} - \log |Q|,$$

$Y \in R^{n \times q}$ has ith row $(y_i - \bar{y})'$, and $X \in R^{n \times p}$ has ith row $(x_i - \bar{x})'$.

Usage

```
mrce(X,Y, lam1=NULL, lam2=NULL, lam1.vec=NULL, lam2.vec=NULL,
     method=c("single", "cv", "fixed.omega"),
     cov.tol=1e-4, cov.maxit=1e3, omega=NULL,
     maxit.out=1e3, maxit.in=1e3, tol.out=1e-8,
     tol.in=1e-8, kfold=5, silent=TRUE, eps=1e-5,
     standardize=FALSE, permute=FALSE)
```

Arguments

`X` An n by p matrix of the values for the prediction variables. The ith row of `X` is x_i defined above ($i = 1, \ldots, n$). Do not include a column of ones.

`Y` An n by q matrix of the observed responses. The ith row of `Y` is y_i defined above ($i = 1, \ldots, n$).

`lam1` A single value for λ_1 defined above. This argument is only used if `method="single"`

`lam2` A single value for λ_2 defined above (or a p by q matrix with (j, k)th entry λ_{2jk} in which case the penalty $2\lambda_2 \sum_{j=1}^{p} \sum_{k=1}^{q} |B_{jk}|$ becomes $2 \sum_{j=1}^{p} \sum_{k=1}^{q} \lambda_{2jk}|B_{jk}|$). This argument is not used if `method="cv"`.

`lam1.vec` A vector of candidate values for λ_1 from which the cross validation procedure searches: only used when `method="cv"` and must be specified by the user when `method="cv"`. Please arrange in decreasing order.

`lam2.vec` A vector of candidate values for λ_2 from which the cross validation procedure searches: only used when `method="cv"` and must be specified by the user when `method="cv"`. Please arrange in decreasing order.

`method` There are three options:

- `method="single"` computes the MRCE estimate of the regression coefficient matrix with penalty tuning parameters `lam1` and `lam2`;
- `method="cv"` performs `kfold` cross validation using candidate tuning parameters in `lam1.vec` and `lam2.vec`;
- `method="fixed.omega"` computes the regression coefficient matrix estimate for which Q (defined above) is fixed at `omega`.

`cov.tol` Convergence tolerance for the glasso algorithm that minimizes the objective function (defined above) with B fixed.

`cov.maxit` The maximum number of iterations allowed for the glasso algorithm that minimizes the objective function (defined above) with B fixed.

`omega` A user-supplied fixed value of Q. Only used when `method="fixed.omega"` in which case the minimizer of the objective function (defined above) with Q fixed at `omega` is returned.

`maxit.out` The maximum number of iterations allowed for the outer loop of the exact MRCE algorithm.

`maxit.in` The maximum number of iterations allowed for the algorithm that minimizes the objective function, defined above, with Ω fixed.

`tol.out` Convergence tolerance for outer loop of the exact MRCE algorithm.

`tol.in` Convergence tolerance for the algorithm that minimizes the objective function, defined above, with Ω fixed.

`kfold` The number of folds to use when `method="cv"`.

`silent` Logical: when `silent=FALSE` this function displays progress updates to the screen.

`eps` The algorithm will terminate if the minimum diagonal entry of the current iterate's residual sample covariance is less than `eps`. This may need adjustment depending on the scales of the variables.

`standardize` Logical: should the columns of `X` be standardized so each has unit length and zero average. The parameter estimates are returned on the original unstandarized scale. The default is `FALSE`.

`permute` Logical: when `method="cv"`, should the subject indices be permutted? The default is `FALSE`.

Details

Please see Rothman, Levina, and Zhu (2010) for more information on the algorithm and model. This version of the software uses the glasso algorithm (Friedman et al., 2008) through the R package `glasso`. If the algorithm is running slowly, track its progress with `silent=FALSE`. In some cases, choosing `cov.tol=0.1` and `tol.out=1e-10` allows the algorithm to make faster progress. If one uses a matrix for `lam2`, consider setting `tol.in=1e-12`.

When $p \geq n$, the diagonal of the optimization variable corresponding to the inverse covariance matrix of the error is penalized. Without diagonal penalization, if there exists a $\bar{B}$ such that the qth column of Y is equal to the qth column of $X\bar{B}$, then a global minimizer of the objective function (defined above) does not exist.

The algorithm that minimizes the objective function, defined above, with Q fixed uses a similar update strategy and termination criterion to those used by Friedman et al. (2010) in the corresponding R package `glmnet`.

Value

A list containing

`Bhat` This is $\hat{\beta} \in R^{p \times q}$ defined above. If `method="cv"`, then `best.lam1` and `best.lam2` defined below are used for λ_1 and λ_2.

`muhat` This is the intercept estimate $\hat{\mu} \in R^q$ defined above. If `method="cv"`, then `best.lam1` and `best.lam2` defined below are used for λ_1 and λ_2.

`omega` This is $\hat{\Omega} \in S_+^q$ defined above. If `method="cv"`, then `best.lam1` and `best.lam2` defined below are used for λ_1 and λ_2.

`mx` This is $\bar{x} \in R^p$ defined above.

`my` This is $\bar{y} \in R^q$ defined above.

`best.lam1` The selected value for λ_1 by cross validation. Will be `NULL` unless `method="cv"`.

`best.lam2` The selected value for λ_2 by cross validation. Will be `NULL` unless `method="cv"`.

`cv.err` Cross validation error matrix with `length(lam1.vec)` rows and `length(lam2.vec)` columns. Will be `NULL` unless `method="cv"`.

Note

The algorithm is fastest when λ_1 and λ_2 are large. Use `silent=FALSE` to check if the algorithm is converging before the total iterations exceeds `maxit.out`.

Author(s)

Adam J. Rothman

References

Rothman, A. J., Levina, E., and Zhu, J. (2010) Sparse multivariate regression with covariance estimation. Journal of Computational and Graphical Statistics. 19: 947–962.

Jerome Friedman, Trevor Hastie, Robert Tibshirani (2008). Sparse inverse covariance estimation with the graphical lasso. Biostatistics, 9(3), 432-441.

Jerome Friedman, Trevor Hastie, Robert Tibshirani (2010). Regularization Paths for Generalized Linear Models via Coordinate Descent. Journal of Statistical Software, 33(1), 1-22.

Examples

```
set.seed(48105)
n=50
p=10
q=5

Omega.inv=diag(q)
for(i in 1:q) for(j in 1:q)
  Omega.inv[i,j]=0.7^abs(i-j)
out=eigen(Omega.inv, symmetric=TRUE)
Omega.inv.sqrt=tcrossprod(out$vec*rep(out$val^(0.5), each=q),out$vec)
Omega=tcrossprod(out$vec*rep(out$val^(-1), each=q),out$vec)

X=matrix(rnorm(n*p), nrow=n, ncol=p)
E=matrix(rnorm(n*q), nrow=n, ncol=q)%*%Omega.inv.sqrt
Beta=matrix(rbinom(p*q, size=1, prob=0.1)*runif(p*q, min=1, max=2), nrow=p, ncol=q)
mu=1:q

Y=rep(1,n)%*%t(mu) + X%*%Beta + E

lam1.vec=rev(10^seq(from=-2, to=0, by=0.5))
lam2.vec=rev(10^seq(from=-2, to=0, by=0.5))
cvfit=mrce(Y=Y, X=X, lam1.vec=lam1.vec, lam2.vec=lam2.vec, method="cv")
cvfit

fit=mrce(Y=Y, X=X, lam1=10^(-1.5), lam2=10^(-0.5), method="single")
fit

lam2.mat=1000*(fit$Bhat==0)
refit=mrce(Y=Y, X=X, lam2=lam2.mat, method="fixed.omega", omega=fit$omega, tol.in=1e-12)
refit
```

stock04 *log-returns of 9 stocks from 2004*

Description

Weekly log-returns of 9 stocks from 2004, analyzed in Yuan et al. (2007)

Usage

```
data(stock04)
```

Format

The format is: num [1:52, 1:9] 0.002275 -0.003795 0.012845 0.017489 -0.000369 ... - attr(*, "dimnames")=List of 2 ..$: NULL ..$: chr [1:9] "Walmart" "Exxon" "GM" "Ford" ...

Source

Yuan, M., Ekici, A., Lu, Z., and Monteiro, R. (2007). Dimension reduction and coefficient estimation in multivariate linear regression. Journal of the Royal Statistical Society Series B, 69(3):329–346.

References

Yuan, M., Ekici, A., Lu, Z., and Monteiro, R. (2007). Dimension reduction and coefficient estimation in multivariate linear regression. Journal of the Royal Statistical Society Series B, 69(3):329–346.

Index