

# Package ‘GulFM’

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**Type** Package

**Title** General Unilateral Load Estimator for Two-Layer Latent Factor Models

**Version** 0.1.2

**Description** Implements general unilateral loading estimator for two-layer latent factor models with smooth, element-wise factor transformations. We provide data simulation, loading estimation, finite-sample error bounds, and diagnostic tools for zero-mean and sub-Gaussian assumptions. A unified interface is given for evaluating estimation accuracy and cosine similarity. The philosophy of the package is described in Guo G. (2026) <[doi:10.1016/j.apm.2025.116280](https://doi.org/10.1016/j.apm.2025.116280)>.

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**Encoding** UTF-8

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**Depends** R (>= 3.5.0)

**Imports** MASS, matrixStats

**Suggests** testthat (>= 3.0.0), ggplot2

**NeedsCompilation** no

**Language** en-US

**Author** Guangbao Guo [aut, cre]

**Maintainer** Guangbao Guo <[ggb11111111@163.com](mailto:ggb11111111@163.com)>

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estimate\_gul\_loadings *General unilateral load Estimator*

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## Description

General unilateral load Estimator

## Usage

```
estimate_gul_loadings(X, m)
```

## Arguments

X                    n \* p data matrix (already centred and scaled if desired).  
m                    number of latent factors (both layers).

## Details

Step 1: PCA on X to get hat\_A1 Step 2: Regress X on hat\_A1 to get hat\_gF1 Step 3: PCA on hat\_gF1 to get hat\_A2 Step 4: hat\_Ag = hat\_A1

## Value

A list with hat\_A1 : p \* m 1st-layer loadings hat\_A2 : m \* m 2nd-layer loadings hat\_Ag : p \* m overall loadings Sigma1 : p \* p sample cov(X) (for diagnostics) Sigma2 : m \* m sample cov(hat\_gF1) hat\_gF1 : n \* m estimated transformed latent factors eig1 : eigen-values of Sigma1 eig2 : eigen-values of Sigma2

## Examples

```
dat <- generate_gfm_data(500, 50, 5, tanh, seed = 1)
est <- estimate_gul_loadings(dat$X, m = 5)
err <- sqrt(mean((est$hat_Ag - dat$Ag)^2)) # overall RMSE
```

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generate_gfm_data	<i>Generate general factor model with smooth latent transformation</i>
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**Description**

Generate general factor model with smooth latent transformation

**Usage**

```
generate_gfm_data(n, p, m, g_fun, seed = 1, sigma_V = 0.1)
```

**Arguments**

n	Integer: sample size.
p	Integer: number of observed variables.
m	Integer: number of latent factors (both layers).
g_fun	Function: smooth, element-wise transformation applied to latent factors. Must be vectorised, e.g. 'sin', 'tanh', 'scale'.
seed	1.
sigma_V	Numeric: standard deviation of the idiosyncratic noise (default 0.1 => Var = 0.01).

**Value**

List with components  $X$  :  $n * p$  matrix of standardised observations.  $A1$  :  $p * m$  first-layer loading matrix.  $A2$  :  $m * m$  second-layer loading matrix.  $Ag$  :  $p * m$  overall loading matrix ( $Ag = A1 F1$  :  $n * m$  latent factors (before transformation).  $gF1$  :  $n * m$  latent factors (after transformation).  $V1$  :  $n * p$  noise matrix (for diagnostics).

**Examples**

```
dat <- generate_gfm_data(200, 50, 5, g_fun = tanh)
```

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gul_simulation	<i>Single-replication GUL simulation</i>
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**Description**

Generates one synthetic data set, estimates loadings with the GUL, and evaluates estimation accuracy.

**Usage**

```
gul_simulation(n, p, m, g_fun)
```

**Arguments**

n	Integer: sample size.
p	Integer: number of observed variables.
m	Integer: number of latent factors (both layers).
g_fun	Function: element-wise, smooth transformation applied to the latent factors (e.g. 'tanh', 'sin').

**Value**

Named numeric vector with components error\_F : Frobenius norm  $\|\hat{A}_g - A_g\|_F$

**Examples**

```
gul_simulation(200, 50, 5, g_fun = tanh)
```

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g_fun	<i>Smooth link functions compliant with Theorems 9&amp;10</i>
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**Description**

Returns a vectorised map  $g(\cdot)$  and its exact Lipschitz constant  $L_g$  for three increasingly nonlinear choices.

**Usage**

```
g_fun(type = c("linear", "weak_nonlinear", "strong_nonlinear"))
```

**Arguments**

type	Character string selecting the map: "linear", "weak_nonlinear", or "strong_nonlinear".
------	--

**Value**

Named list with components

g_fun	vectorised function $g(\cdot)$
L_g	scalar Lipschitz constant of $g$

**Examples**

```
## pick a link with L_g = 1
tmp <- g_fun("linear")
dat <- generate_gfm_data(n = 500, p = 200, m = 5, g_fun = tmp$g_fun)
est <- estimate_gul_loadings(dat$X, m = 5)
err <- norm(est$hat_Ag - dat$Ag, "F")
sprintf("F-error (L_g = %d) = %.3f", tmp$L_g, err)
```

g\_theorem

*Simulation wrapper for Theorems 9 & 10***Description**

One Monte-Carlo replicate; returns empirical error, exceedance indicator, theoretical bounds, and assumption-check flags.

**Usage**

```
g_theorem(n, p, m, g_type, epsilon, zero_tol = 0.02)
```

**Arguments**

n	sample size
p	number of observed variables
m	number of latent factors
g_type	character: "linear", "weak_nonlinear", "strong_nonlinear"
epsilon	error threshold
zero_tol	zero-mean tolerance (default 0.02)

**Value**

one-row data-frame

**Examples**

```
df <- g_theorem(500, 200, 5, "linear", 0.6)
```

loading\_metrics

*Multi-metric evaluation of factor loading matrix estimation error***Description**

Multi-metric evaluation of factor loading matrix estimation error

**Usage**

```
loading_metrics(A_true, A_hat)
```

**Arguments**

A_true	True loading matrix (p x m)
A_hat	Estimated loading matrix (p x m)

**Value**

data.frame with MSE, RMSE, MAE, MaxDev, and Cosine similarity

**Examples**

```
## simulated example
p <- 100; m <- 5
Ag_true <- matrix(rnorm(p*m), p, m)
Ag_hat <- Ag_true + matrix(rnorm(p*m, 0, 0.1), p, m)
metrics <- loading_metrics(Ag_true, Ag_hat)
print(metrics)
```

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verify\_mean

*Verify zero-mean preservation (Theorem 10 assumption 2a)*

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**Description**

Draws  $n$  i.i.d.  $N(0, I_m)$  latent factors, applies  $g$  component-wise, and checks whether  $|E[g(x)]| < \text{tol}$  on every coordinate.

**Usage**

```
verify_mean(g_fun, m = 5, n = 10000, tol = 0.001)
```

**Arguments**

<code>g_fun</code>	vectorised map $g: \mathbb{R} \rightarrow \mathbb{R}$
<code>m</code>	latent dimension
<code>n</code>	Monte-Carlo sample size
<code>tol</code>	numerical tolerance (default $1e-3$ )

**Value**

logical TRUE if  $|\text{lmean}| < \text{tol}$  on all coords

**Examples**

```
tmp <- g_fun("weak_nonlinear")
verify_mean(tmp$g_fun, m = 5)
```

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verify\_subgaussian      *Verify sub-Gaussian preservation*

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**Description**

Draws  $n$  i.i.d.  $N(0, I_m)$  latent factors, applies  $g$  component-wise, and checks whether  $E[\exp(g(x))]$  remains below an empirical cut-off. This is a quick proxy for finite sub-Gaussian norm.

**Usage**

```
verify_subgaussian(g_fun, m = 5, n = 1000, cut = exp(2))
```

**Arguments**

<code>g_fun</code>	vectorised map $g: \mathbb{R} \rightarrow \mathbb{R}$
<code>m</code>	latent dimension
<code>n</code>	Monte-Carlo sample size
<code>cut</code>	empirical threshold (default $\exp(2)$ & 7.389)

**Value**

logical TRUE if  $E[\exp(g)] < \text{cut}$  on all coords

**Examples**

```
tmp <- g_fun("strong_nonlinear")
verify_subgaussian(tmp$g_fun, m = 5)
```

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