Package ‘CompQuadForm’

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Title Distribution Function of Quadratic Forms in Normal Variables

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Description Computes the distribution function of quadratic forms in normal variables using Imhof's method, Davies's algorithm, Farebrother's algorithm or Liu et al.'s algorithm.

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Contents

`davies` *Davies method*

Description

Distribution function (survival function in fact) of quadratic forms in normal variables using Davies's method.

Usage

```
davies(q, lambda, h = rep(1, length(lambda)), delta = rep(0,
         length(lambda)), sigma = 0, lim = 10000, acc = 0.0001)
```

Arguments

`q`	value point at which distribution function is to be evaluated
`lambda`	the weights $\lambda_1, \lambda_2, ..., \lambda_n$, i.e. distinct non-zero characteristic roots of $A\Sigma$
`h`	respective orders of multiplicity n_j of the λs
`delta`	non-centrality parameters δ^2_j (should be positive)
`sigma`	coefficient σ of the standard Gaussian
`lim`	maximum number of integration terms. Realistic values for 'lim' range from 1,000 if the procedure is to be called repeatedly up to 50,000 if it is to be called only occasionally
`acc`	error bound. Suitable values for 'acc' range from 0.001 to 0.00005 which should be adequate for most statistical purposes.

Details

Computes $P[Q > q]$ where $Q = \sum_{j=1}^{r} \lambda_j X_j + \sigma X_0$ where X_j are independent random variables having a non-central chi^2 distribution with n_j degrees of freedom and non-centrality parameter $delta^2_j$ for $j = 1, ..., r$ and X_0 having a standard Gaussian distribution.

Value

`trace`	vector, indicating performance of procedure, with the following components: 1: absolute value sum, 2: total number of integration terms, 3: number of integrations, 4: integration interval in main integration, 5: truncation point in initial integration, 6: standard deviation of convergence factor term, 7: number of cycles to locate integration parameters
`ifault`	fault indicator: 0: no error, 1: requested accuracy could not be obtained, 2: round-off error possibly significant, 3: invalid parameters, 4: unable to locate integration parameters
`Qq`	$P[Q > q]$

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References

P. Duchesne, P. Lafaye de Micheaux, Computing the distribution of quadratic forms: Further comparisons between the Liu-Tang-Zhang approximation and exact methods, *Computational Statistics and Data Analysis*, Volume 54, (2010), 858-862

Davies R.B., Algorithm AS 155: The Distribution of a Linear Combination of chi-2 Random Variables, Journal of the Royal Statistical Society. Series C (Applied Statistics), 29(3), p. 323-333, (1980)

Examples

```
# Some results from Table 3, p.327, Davies (1980)

 round(1 - davies(1, c(6, 3, 1), c(1, 1, 1))$Qq, 4)
 round(1 - davies(7, c(6, 3, 1), c(1, 1, 1))$Qq, 4)
 round(1 - davies(20, c(6, 3, 1), c(1, 1, 1))$Qq, 4)

 round(1 - davies(2, c(6, 3, 1), c(2, 2, 2))$Qq, 4)
 round(1 - davies(20, c(6, 3, 1), c(2, 2, 2))$Qq, 4)
 round(1 - davies(60, c(6, 3, 1), c(2, 2, 2))$Qq, 4)

 round(1 - davies(10, c(6, 3, 1), c(6, 4, 2))$Qq, 4)
 round(1 - davies(50, c(6, 3, 1), c(6, 4, 2))$Qq, 4)
 round(1 - davies(120, c(6, 3, 1), c(6, 4, 2))$Qq, 4)

 round(1 - davies(20, c(7, 3), c(6, 2), c(6, 2))$Qq, 4)
 round(1 - davies(100, c(7, 3), c(6, 2), c(6, 2))$Qq, 4)
 round(1 - davies(200, c(7, 3), c(6, 2), c(6, 2))$Qq, 4)

 round(1 - davies(10, c(7, 3), c(1, 1), c(6, 2))$Qq, 4)
 round(1 - davies(60, c(7, 3), c(1, 1), c(6, 2))$Qq, 4)
 round(1 - davies(150, c(7, 3), c(1, 1), c(6, 2))$Qq, 4)

 round(1 - davies(70, c(7, 3, 7, 3), c(6, 2, 1, 1), c(6, 2, 6, 2))$Qq, 4)
 round(1 - davies(160, c(7, 3, 7, 3), c(6, 2, 1, 1), c(6, 2, 6, 2))$Qq, 4)
 round(1 - davies(260, c(7, 3, 7, 3), c(6, 2, 1, 1), c(6, 2, 6, 2))$Qq, 4)

 round(1 - davies(-40, c(7, 3, -7, -3), c(6, 2, 1, 1), c(6, 2, 6,
 2))$Qq, 4)
 round(1 - davies(40, c(7, 3, -7, -3), c(6, 2, 1, 1), c(6, 2, 6, 2))$Qq,
 4)
 round(1 - davies(140, c(7, 3, -7, -3), c(6, 2, 1, 1), c(6, 2, 6,
 2))$Qq, 4)

# You should sometimes play with the 'lim' parameter:
davies(0.00001,lambda=0.2)
imhof(0.00001,lambda=0.2)$Qq
davies(0.00001,lambda=0.2, lim=20000)
```

farebrother *Ruben/Farebrother method*

Description

Distribution function (survival function in fact) of quadratic forms in normal variables using Farebrother's algorithm.

Usage

```
farebrother(q, lambda, h = rep(1, length(lambda)),
            delta = rep(0, length(lambda)), maxit = 100000,
            eps = 10^(-10), mode = 1)
```

Arguments

`q`	value point at which distribution function is to be evaluated
`lambda`	the weights $\lambda_1, \lambda_2, ..., \lambda_n$, i.e. the distinct non-zero characteristic roots of $A\Sigma$
`h`	vector of the respective orders of multiplicity m_i of the λs
`delta`	the non-centrality parameters δ_i (should be positive)
`maxit`	the maximum number of term K in equation below
`eps`	the desired level of accuracy
`mode`	if 'mode' > 0 then $\beta = mode * \lambda_{min}$ otherwise $\beta = \beta_B = 2/(1/\lambda_{min} + 1/\lambda_{max})$

Details

Computes P[Q>q] where $Q = \sum_{j=1}^{n} \lambda_j \chi^2(m_j, \delta_j^2)$. P[Q<q] is approximated by $\sum_k = 0^{K-1} a_k P[\chi^2(m+2k) < q/\beta]$ where $m = \sum_{j=1}^{n} m_j$ and β is an arbitrary constant (as given by argument mode).

Value

`dnsty`	the density of the linear form
`ifault`	the fault indicator. -i: one or more of the constraints $\lambda_i > 0$

, $m_i > 0$ and $\delta_i^2 \geq 0$ is not satisfied. 1: non-fatal underflow of a_0. 2: one or more of the constraints $n > 0$, $q > 0$, $maxit > 0$ and $eps > 0$ is not satisfied. 3: the current estimate of the probability is greater than 2. 4: the required accuracy could not be obtained in 'maxit' iterations. 5: the value returned by the procedure does not satisfy $0 \leq RUBEN \leq 1$. 6: 'dnsty' is negative. 9: faults 4 and 5. 10: faults 4 and 6. 0: otherwise.

`Qq`	$P[Q > q]$

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References

P. Duchesne, P. Lafaye de Micheaux, Computing the distribution of quadratic forms: Further comparisons between the Liu-Tang-Zhang approximation and exact methods, *Computational Statistics and Data Analysis*, Volume 54, (2010), 858-862

Farebrother R.W., Algorithm AS 204: The distribution of a Positive Linear Combination of chi-squared random variables, Journal of the Royal Statistical Society, Series C (applied Statistics), Vol. 33, No. 3 (1984), p. 332-339

Examples

```
# Some results from Table 3, p.327, Davies (1980)

 1 - farebrother(1, c(6, 3, 1), c(1, 1, 1), c(0, 0, 0))$Qq
```

`imhof` *Imhof method.*

Description

Distribution function (survival function in fact) of quadratic forms in normal variables using Imhof's method.

Usage

```
imhof(q, lambda, h = rep(1, length(lambda)),
      delta = rep(0, length(lambda)),
      epsabs = 10^(-6), epsrel = 10^(-6), limit = 10000)
```

Arguments

q	value point at which the survival function is to be evaluated
lambda	distinct non-zero characteristic roots of $A\Sigma$
h	respective orders of multiplicity of the λs
delta	non-centrality parameters (should be positive)
epsabs	absolute accuracy requested
epsrel	relative accuracy requested
limit	determines the maximum number of subintervals in the partition of the given integration interval

Details

Let $\boldsymbol{X} = (X_1, \ldots, X_n)'$ be a column random vector which follows a multidimensional normal law with mean vector $\mathbf{0}$ and non-singular covariance matrix $\boldsymbol{\Sigma}$. Let $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_n)'$ be a constant vector, and consider the quadratic form

$$Q = (\boldsymbol{x} + \boldsymbol{\mu})' \boldsymbol{A} (\boldsymbol{x} + \boldsymbol{\mu}) = \sum_{r=1}^{m} \lambda_r \chi^2_{h_r;\delta_r}.$$

The function `imhof` computes $P[Q > q]$.

The λ_r's are the distinct non-zero characteristic roots of $A\Sigma$, the h_r's their respective orders of multiplicity, the δ_r's are certain linear combinations of $\mu_1, \ldots, \mu_n$ and the $\chi^2_{h_r;\delta_r}$ are independent χ^2-variables with h_r degrees of freedom and non-centrality parameter δ_r. The variable $\chi^2_{h,\delta}$ is defined here by the relation $\chi^2_{h,\delta} = (X_1 + \delta)^2 + \sum_{i=2}^{h} X_i^2$, where $X_1, \ldots, X_h$ are independent unit normal deviates.

Value

`Qq`	$P[Q > q]$
`abserr`	estimate of the modulus of the absolute error, which should equal or exceed abs(i - result)

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References

P. Duchesne, P. Lafaye de Micheaux, Computing the distribution of quadratic forms: Further comparisons between the Liu-Tang-Zhang approximation and exact methods, *Computational Statistics and Data Analysis*, Volume 54, (2010), 858-862

J. P. Imhof, Computing the Distribution of Quadratic Forms in Normal Variables, *Biometrika*, Volume 48, Issue 3/4 (Dec., 1961), 419-426

Examples

```
# Some results from Table 1, p.424, Imhof (1961)

# Q1 with x = 2
round(imhof(2, c(0.6, 0.3, 0.1))$Qq, 4)

# Q2 with x = 6
round(imhof(6, c(0.6, 0.3, 0.1), c(2, 2, 2))$Qq, 4)

# Q6 with x = 15
round(imhof(15, c(0.7, 0.3), c(1, 1), c(6, 2))$Qq, 4)
```

`liu` *Liu's method*

Description

Distribution function (survival function in fact) of quadratic forms in normal variables using Liu et al.'s method.

Usage

```
liu(q, lambda, h = rep(1, length(lambda)),
    delta = rep(0, length(lambda)))
```

Arguments

`q`	value point at which the survival function is to be evaluated
`lambda`	distinct non-zero characteristic roots of $A\Sigma$, i.e. the λ_i's
`h`	respective orders of multiplicity h_i's of the λ's
`delta`	non-centrality parameters δ_i's (should be positive)

Details

New chi-square approximation to the distribution of non-negative definite quadratic forms in non-central normal variables.

Computes $P[Q > q]$ where $Q = \sum_{j=1}^{n} \lambda_j \chi^2(h_j, \delta_j)$.

This method does not work as good as the Imhof's method. Thus Imhof's method should be recommended.

Value

`Qq`	$P[Q > q]$

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References

P. Duchesne, P. Lafaye de Micheaux, Computing the distribution of quadratic forms: Further comparisons between the Liu-Tang-Zhang approximation and exact methods, *Computational Statistics and Data Analysis*, Volume 54, (2010), 858-862

H. Liu, Y. Tang, H.H. Zhang, A new chi-square approximation to the distribution of non-negative definite quadratic forms in non-central normal variables, *Computational Statistics and Data Analysis*, Volume 53, (2009), 853-856

Examples

```
# Some results from Liu et al. (2009)
# Q1 from Liu et al.
round(liu(2, c(0.5, 0.4, 0.1), c(1, 2, 1), c(1, 0.6, 0.8)), 6)
round(liu(6, c(0.5, 0.4, 0.1), c(1, 2, 1), c(1, 0.6, 0.8)), 6)
round(liu(8, c(0.5, 0.4, 0.1), c(1, 2, 1), c(1, 0.6, 0.8)), 6)

# Q2 from Liu et al.
round(liu(1, c(0.7, 0.3), c(1, 1), c(6, 2)), 6)
round(liu(6, c(0.7, 0.3), c(1, 1), c(6, 2)), 6)
round(liu(15, c(0.7, 0.3), c(1, 1), c(6, 2)), 6)

# Q3 from Liu et al.
round(liu(2, c(0.995, 0.005), c(1, 2), c(1, 1)), 6)
round(liu(8, c(0.995, 0.005), c(1, 2), c(1, 1)), 6)
round(liu(12, c(0.995, 0.005), c(1, 2), c(1, 1)), 6)

# Q4 from Liu et al.
round(liu(3.5, c(0.35, 0.15, 0.35, 0.15), c(1, 1, 6, 2), c(6, 2, 6, 2)),
6)
round(liu(8, c(0.35, 0.15, 0.35, 0.15), c(1, 1, 6, 2), c(6, 2, 6, 2)), 6)
round(liu(13, c(0.35, 0.15, 0.35, 0.15), c(1, 1, 6, 2), c(6, 2, 6, 2)), 6)
```

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