

# Package ‘Carlson’

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**Type** Package

**Title** Carlson Elliptic Integrals and Incomplete Elliptic Integrals

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**Description** Evaluation of the Carlson elliptic integrals and the incomplete elliptic integrals with complex arguments. The implementations use Carlson's algorithms <[doi:10.1007/BF02198293](https://doi.org/10.1007/BF02198293)>. Applications of elliptic integrals include probability distributions, geometry, physics, mechanics, electrodynamics, statistical mechanics, astronomy, geodesy, geodesics on conics, and magnetic field calculations.

**License** GPL-3

**URL** <https://github.com/stla/Carlson>

**BugReports** <https://github.com/stla/Carlson/issues>

**Imports** Rcpp

**LinkingTo** Rcpp

**Suggests** gsl, testthat

**Encoding** UTF-8

**RoxygenNote** 7.2.3

**NeedsCompilation** yes

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Carlson_RC	<i>Carlson elliptic integral RC</i>
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## Description

Evaluate the Carlson elliptic integral RC.

## Usage

```
Carlson_RC(x, y, minerror = 1e-15)
```

## Arguments

x, y	real or complex numbers, with y different from 0
minerror	bound on the relative error passed to <a href="#">Carlson_RF</a>

## Value

A complex number, the value of the Carlson elliptic integral  $R_C(x, y)$ .

## Note

The function returns a value when x or y are negative real numbers, but this value is not the one of the Carlson integral.

## Examples

```
Carlson_RC(5, 2)
gsl::ellint_RC(5, 2)
```

---

Carlson\_RD

*Carlson elliptic integral RD*

---

### Description

Evaluate the Carlson elliptic integral RD.

### Usage

```
Carlson_RD(x, y, z, minerror = 1e-15)
```

### Arguments

x, y, z	real or complex numbers; at most one can be 0
minerror	bound on the relative error

### Value

A complex number, the value of the Carlson elliptic integral  $R_D(x, y, z)$ .

### Note

The function returns a value when x, y or z are negative real numbers, but this value is not the one of the Carlson integral.

### Examples

```
Carlson_RD(5, 2, 3)  
gsl::ellint_RD(5, 2, 3)
```

---

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Carlson\_RF

*Carlson elliptic integral RF*

---

### Description

Evaluate the Carlson elliptic integral RF.

### Usage

```
Carlson_RF(x, y, z, minerror = 1e-15)
```

### Arguments

x, y, z	real or complex numbers; at most one can be 0
minerror	bound on relative error

**Value**

A complex number, the value of the Carlson elliptic integral  $R_F(x, y, z)$ .

**Note**

The function returns a value when  $x, y$  or  $z$  are negative real numbers, but this value is not the one of the Carlson integral.

**Examples**

```
Carlson_RF(5, 2, 3)
gsl::ellint_RF(5, 2, 3)
```

Carlson\_RG

*Carlson elliptic integral RG***Description**

Evaluate the Carlson elliptic integral RG.

**Usage**

```
Carlson_RG(x, y, z, minerror = 1e-15)
```

**Arguments**

$x, y, z$	real or complex numbers; they can be zero
minerror	bound on the relative error passed to <a href="#">Carlson_RF</a> and <a href="#">Carlson_RD</a>

**Value**

A complex number, the value of the Carlson elliptic integral  $R_G(x, y, z)$ .

Carlson\_RJ

*Carlson elliptic integral RJ***Description**

Evaluate the Carlson elliptic integral RJ.

**Usage**

```
Carlson_RJ(x, y, z, p, minerror = 1e-15)
```

**Arguments**

x, y, z, p	real or complex numbers; at most one can be 0
minerror	bound on the relative error

**Value**

A complex number, the value of the Carlson elliptic integral  $R_J(x, y, z, t)$ .

**Note**

The function returns a value when x, y, z or p are negative real numbers, but this value is not the one of the Carlson integral.

**Examples**

```
Carlson_RJ(5, 2, 3, 4)
gsl::ellint_RJ(5, 2, 3, 4)
```

**elliptic\_E**

*Incomplete elliptic integral of the second kind*

**Description**

Evaluate the incomplete elliptic integral of the second kind.

**Usage**

```
elliptic_E(phi, m, minerror = 1e-15)
```

**Arguments**

phi	amplitude, real or complex number/vector
m	parameter, real or complex number/vector
minerror	the bound on the relative error passed to <a href="#">Carlson_RF</a> and <a href="#">Carlson_RD</a>

**Value**

A complex number or vector, the value(s) of the incomplete elliptic integral  $E(\phi, m)$ .

**Examples**

```
elliptic_E(1, 0.2)
gsl::ellint_E(1, sqrt(0.2))
```

**elliptic\_F***Incomplete elliptic integral of the first kind***Description**

Evaluate the incomplete elliptic integral of the first kind.

**Usage**

```
elliptic_F(phi, m, minerror = 1e-15)
```

**Arguments**

phi	amplitude, real or complex number/vector
m	parameter, real or complex number/vector
minerror	the bound on the relative error passed to <a href="#">Carlson_RF</a>

**Value**

A complex number or vector, the value(s) of the incomplete elliptic integral  $F(\phi, m)$ .

**Examples**

```
elliptic_F(1, 0.2)
gsl::ellint_F(1, sqrt(0.2))
```

**elliptic\_PI***Incomplete elliptic integral of the third kind***Description**

Evaluate the incomplete elliptic integral of the third kind.

**Usage**

```
elliptic_PI(phi, n, m, minerror = 1e-15)
```

**Arguments**

phi	amplitude, real or complex number/vector
n	characteristic, real or complex number/vector
m	parameter, real or complex number/vector
minerror	the bound on the relative error passed to <a href="#">Carlson_RF</a> and <a href="#">Carlson_RJ</a>

**Value**

A complex number or vector, the value(s) of the incomplete elliptic integral  $\Pi(\phi, n, m)$ .

**Examples**

```
elliptic_PI(1, 0.8, 0.2)
gsl::ellint_P(1, sqrt(0.2), -0.8)
```

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elliptic\_Z

*Jacobi zeta function*

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**Description**

Evaluate the Jacobi zeta function.

**Usage**

```
elliptic_Z(phi, m, minerror = 1e-15)
```

**Arguments**

phi	amplitude, real or complex number/vector
m	parameter, real or complex number/vector
minerror	bound on relative error passed to <a href="#">elliptic_E</a> and <a href="#">elliptic_F</a>

**Value**

A complex number or vector, the value(s) of the Jacobi zeta function  $Z(\phi, m)$ .

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Lambda0

*Heuman Lambda function*

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**Description**

Evaluates the Heuman Lambda function.

**Usage**

```
Lambda0(phi, m, minerror = 1e-14)
```

**Arguments**

phi	Jacobi amplitude, a complex number/vector
m	parameter, a complex number/vector
minerror	the bound on the relative error passed to <a href="#">elliptic_F</a> and <a href="#">elliptic_Z</a>

**Value**

A complex number or vector.

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