Package ‘Bivariate.Pareto’

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Type Package

Title Bivariate Pareto Models

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Description Perform competing risks analysis under bivariate Pareto models. See Shih et al. (2019) <doi:10.1080/03610926.2018.1425450> for details.

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Contents

`Bivariate.Pareto-package`
Bivariate Pareto Models

Description

Perform competing risks analysis under bivariate Pareto models. See Shih et al. (2018) for details.

Details

The functions in this package are based on latent failure time models with competing risks in Shih et al. (2018). However, they can be adapted to dependent censoring models in Emura and Chen (2018). See `MLE.SN.Pareto` for example.

Author(s)

Jia-Han Shih, Wei Lee

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References

Shih J-H, Lee W, Sun L-H, Emura T (2018), Fitting competing risks data to bivariate Pareto models, Communications in Statistics - Theory and Methods, doi: 10.1080/03610926.2018.1425450.

Emura T, Chen Y-H (2018) Analysis of Survival Data with Dependent Censoring, Copula-Based Approaches, JSS Research Series in Statistics, Springer, in press.

`Frank.Pareto` *Generate samples from the Frank copula with the Pareto margins*

Description

Generate samples from the Frank copula with the Pareto margins.

Usage

```
Frank.Pareto(n, Theta, Alpha1, Alpha2, Gamma1, Gamma2)
```

Arguments

`n`	Sample size.
`Theta`	Copula parameter θ.
`Alpha1`	Positive scale parameter α_1 for the Pareto margin.
`Alpha2`	Positive scale parameter α_2 for the Pareto margin.
`Gamma1`	Positive shape parameter γ_1 for the Pareto margin.
`Gamma2`	Positive shape parameter γ_2 for the Pareto margin.

Value

X	X is asscoiated with the parameters Alpha1 and Gamma1.
Y	Y is asscoiated with the parameters Alpha2 and Gamma2.

References

Shih J-H, Lee W, Sun L-H, Emura T (2019), Fitting competing risks data to bivariate Pareto models, Communications in Statistics - Theory and Methods, 48:1193-1220.

Examples

```
library(Bivariate.Pareto)
Frank.Pareto(5,5,1,1,1,1)
```

Kendall.SNBP — *Kendall's tau under the SNBP distribution*

Description

Compute Kendall's tau under the Sankaran and Nair bivairate Pareto (SNBP) distribution (Sankaran and Nair, 1993) by numerical integration.

Usage

```
Kendall.SNBP(Alpha0, Alpha1, Alpha2, Gamma)
```

Arguments

Alpha0	Copula parameter α_0 with restricted range.
Alpha1	Positive scale parameter α_1 for the Pareto margin.
Alpha2	Positive scale parameter α_2 for the Pareto margin.
Gamma	Common positive shape parameter γ for the Pareto margins.

Details

The admissible range of Alpha0 (α_0) is $0 \leq \alpha_0 \leq (\gamma+1)\alpha_1\alpha_2$.

Value

tau	Kendall's tau.

References

Sankaran PG, Nair NU (1993), A bivariate Pareto model and its applications to reliability, Naval Research Logistics, 40:1013-1020.

Shih J-H, Lee W, Sun L-H, Emura T (2019), Fitting competing risks data to bivariate Pareto models, Communications in Statistics - Theory and Methods, 48:1193-1220.

Examples

```
library(Bivariate.Pareto)
Kendall.SNBP(7e-5,0.0036,0.0075,1.8277)
```

MLE.Frank.Pareto *Maximum likelihood estimation for bivariate dependent competing risks data under the Frank copula with the Pareto margins and fixed θ*

Description

Maximum likelihood estimation for bivariate dependent competing risks data under the Frank copula with the Pareto margins and fixed θ.

Usage

```
MLE.Frank.Pareto(
  t.event,
  event1,
  event2,
  Theta,
  Alpha1.0 = 1,
  Alpha2.0 = 1,
  Gamma1.0 = 1,
  Gamma2.0 = 1,
  epsilon = 1e-05,
  d = exp(10),
  r.1 = 6,
  r.2 = 6,
  r.3 = 6,
  r.4 = 6
)
```

Arguments

t.event	Vector of the observed failure times.
event1	Vector of the indicators for the failure cause 1.
event2	Vector of the indicators for the failure cause 2.
Theta	Copula parameter θ.
Alpha1.0	Initial guess for the scale parameter α_1 with default value 1.
Alpha2.0	Initial guess for the scale parameter α_2 with default value 1.
Gamma1.0	Initial guess for the shape parameter γ_1 with default value 1.
Gamma2.0	Initial guess for the shape parameter γ_2 with default value 1.
epsilon	Positive tunning parameter in the NR algorithm with default value 10^{-5}.
d	Positive tunning parameter in the NR algorithm with default value e^{10}.

`r.1`	Positive tunning parameter in the NR algorithm with default value 1.
`r.2`	Positive tunning parameter in the NR algorithm with default value 1.
`r.3`	Positive tunning parameter in the NR algorithm with default value 1.
`r.4`	Positive tunning parameter in the NR algorithm with default value 1.

Value

`n`	Sample size.
`count`	Iteration number.
`random`	Randomization number.
`Alpha1`	Positive scale parameter for the Pareto margin (failure cause 1).
`Alpha2`	Positive scale parameter for the Pareto margin (failure cause 2).
`Gamma1`	Positive shape parameter for the Pareto margin (failure cause 1).
`Gamma2`	Positive shape parameter for the Pareto margin (failure cause 2).
`MedX`	Median lifetime due to failure cause 1.
`MedY`	Median lifetime due to failure cause 2.
`MeanX`	Mean lifetime due to failure cause 1.
`MeanY`	Mean lifetime due to failure cause 2.
`logL`	Log-likelihood value under the fitted model.
`AIC`	AIC value under the fitted model.
`BIC`	BIC value under the fitted model.

References

Shih J-H, Lee W, Sun L-H, Emura T (2018), Fitting competing risks data to bivariate Pareto models, Communications in Statistics - Theory and Methods, doi: 10.1080/03610926.2018.1425450.

Examples

```
t.event = c(72,40,20,65,24,46,62,61,60,60,59,59,49,20, 3,58,29,26,52,20,
            51,51,31,42,38,69,39,33, 8,13,33, 9,21,66, 5,27, 2,20,19,60,
            32,53,53,43,21,74,72,14,33, 8,10,51, 7,33, 3,43,37, 5, 6, 2,
            5,64, 1,21,16,21,12,75,74,54,73,36,59, 6,58,16,19,39,26,60,
            43, 7, 9,67,62,17,25, 0, 5,34,59,31,58,30,57, 5,55,55,52, 0,
            51,17,70,74,74,20, 2, 8,27,23, 1,52,51, 6, 0,26,65,26, 6, 6,
            68,33,67,23, 6,11, 6,57,57,29, 9,53,51, 8, 0,21,27,22,12,68,
            21,68, 0, 2,14,18, 5,60,40,51,50,46,65, 9,21,27,54,52,75,30,
            70,14, 0,42,12,40, 2,12,53,11,18,13,45, 8,28,67,67,24,64,26,
            57,32,42,20,71,54,64,51, 1, 2, 0,54,69,68,67,66,64,63,35,62,
            7,35,24,57, 1, 4,74, 0,51,36,16,32,68,17,66,65,19,41,28, 0,
            46,63,60,59,46,63, 8,74,18,33,12, 1,66,28,30,57,50,39,40,24,
            6,30,58,68,24,33,65, 2,64,19,15,10,12,53,51, 1,40,40,66, 2,
            21,35,29,54,37,10,29,71,12,13,27,66,28,31,12, 9,21,19,51,71,
            76,46,47,75,75,49,75,75,31,69,74,25,72,28,36, 8,71,60,14,22,
            67,62,68,68,27,68,68,67,67, 3,49,12,30,67, 5,65,24,66,36,66,
            40,13,40, 0,14,45,64,13,24,15,26, 5,63,35,61,61,50,57,21,26,
```

```
                 11,59,42,27,50,57,57, 0, 1,54,53,23, 8,51,27,52,52,52,45,48,
                 18, 2, 2,35,75,75, 9,39, 0,26,17,43,53,47,11,65,16,21,64, 7,
                 38,55, 5,28,38,20,24,27,31, 9, 9,11,56,36,56,15,51,33,70,32,
                 5,23,63,30,53,12,58,54,36,20,74,34,70,25,65, 4,10,58,37,56,
                 6, 0,70,70,28,40,67,36,23,23,62,62,62, 2,34, 4,12,56, 1, 7,
                 4,70,65, 7,30,40,13,22, 0,18,64,13,26, 1,16,33,22,30,53,53,
                 7,61,40, 9,59, 7,12,46,50, 0,52,19,52,51,51,14,27,51, 5, 0,
                 41,53,19)

event1 = c(0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,
           0,0,1,0,0,0,1,0,1,1,0,1,1,1,1,0,0,1,1,0,
           1,0,0,1,1,0,0,1,0,0,0,1,0,1,0,0,1,0,1,1,
           1,0,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,1,1,0,0,0,0,0,1,1,0,0,1,0,0,
           0,0,0,1,0,0,1,0,0,0,0,0,0,0,0,0,1,0,1,0,
           0,0,0,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,1,1,0,1,0,0,0,0,1,0,0,0,0,0,
           1,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,0,1,0,0,1,1,0,1,0,0,1,1,0,0,
           1,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,1,0,0,0,
           0,0,1,0,1,0,0,0,0,1,1,1,1,0,0,0,1,1,0,0,
           1,1,1,1,0,0,1,0,1,1,1,1,1,1,1,0,1,1,0,1,
           0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,
           0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,
           0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,
           1,0,0,0,0,0,0,1,0,0,0,0,1,0,1,0,1,0,0,1,
           1,1,0,1,1,1,1,1,1,1,1,0,1,1,0,0,0,0,0,0,
           0,0,0,1,0,0,0,0,1,0,0,1,0,1,0,1,1,0,1,0,
           1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,
           1,0,0,1,0,0,0,1,0,1,0,0,1,0,0,0,1,1,0,1,
           1,1,1,0,0,0,1,0,0,0,0,0,0,0,0,1,1,0,0,0,
           0,0,1)

event2 = c(0,1,1,0,0,1,0,0,0,0,0,0,0,1,1,0,1,1,0,1,
           0,0,0,1,1,0,0,1,0,0,1,0,0,0,0,1,1,0,0,0,
           0,0,0,0,0,0,0,0,1,1,1,0,1,0,1,1,0,1,0,0,
           0,0,1,0,1,1,1,0,0,0,0,1,1,1,1,1,1,1,1,1,
           1,1,1,0,1,1,1,1,1,1,0,1,0,1,0,1,0,0,0,1,
           0,1,1,0,0,1,0,0,1,1,1,0,0,0,0,1,1,0,1,1,
           0,1,0,0,1,1,0,0,0,1,1,0,0,1,1,1,0,1,0,0,
           1,0,1,0,0,1,0,0,1,0,1,1,0,1,1,1,0,0,0,1,
           0,1,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,1,0,1,
           0,0,1,1,0,1,0,1,1,1,0,1,0,0,0,0,0,0,1,0,
           1,1,1,0,1,1,1,0,1,1,0,0,0,0,0,0,0,0,1,1,
           0,0,0,0,1,0,1,0,1,1,1,1,0,1,1,1,0,1,1,1,
           1,1,0,0,0,1,0,1,0,0,0,0,0,0,0,1,0,0,0,1,
           0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,
           0,0,1,0,0,1,0,0,1,0,0,1,0,1,1,0,0,1,1,1,
           1,1,0,0,1,0,0,0,0,1,1,1,1,0,1,1,1,0,1,0,
           1,1,1,1,1,1,0,1,1,1,1,0,0,1,0,0,1,1,1,0,
           1,0,0,1,1,0,0,1,1,0,0,1,1,1,1,0,0,0,1,1,
```

```
          0,1,1,1,0,0,1,0,1,1,1,1,0,1,0,0,0,1,0,0,
          0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,1,0,1,
          1,1,0,0,1,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,
          0,1,0,0,1,1,0,1,1,1,0,0,0,1,0,1,0,0,1,1,
          0,0,0,0,1,1,1,0,1,0,1,1,0,1,1,1,0,0,1,0,
          0,0,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0,0,1,1,
          1,0,0)

library(Bivariate.Pareto)
set.seed(10)
MLE.Frank.Pareto(t.event,event1,event2,Theta = -5)
```

MLE.Frank.Pareto.com *Maximum likelihood estimation for bivariate dependent competing risks data under the Frank copula with the common Pareto margins*

Description

Maximum likelihood estimation for bivariate dependent competing risks data under the Frank copula with the common Pareto margins.

Usage

```
MLE.Frank.Pareto.com(
  t.event,
  event1,
  event2,
  Theta.0 = 1,
  Alpha.0 = 1,
  Gamma.0 = 1,
  epsilon = 1e-05,
  r.1 = 13,
  r.2 = 3,
  r.3 = 3,
  bootstrap = FALSE,
  B = 200
)
```

Arguments

`t.event`	Vector of the observed failure times.
`event1`	Vector of the indicators for the failure cause 1.
`event2`	Vector of the indicators for the failure cause 2.
`Theta.0`	Initial guess for the copula parameter θ.
`Alpha.0`	Initial guess for the common scale parameter α with default value 1.
`Gamma.0`	Initial guess for the common shape parameter γ with default value 1.

epsilon	Positive tunning parameter in the NR algorithm with default value 10^{-5}.
r.1	Positive tunning parameter in the NR algorithm with default value 1.
r.2	Positive tunning parameter in the NR algorithm with default value 1.
r.3	Positive tunning parameter in the NR algorithm with default value 1.
bootstrap	Perform parametric bootstrap if TRUE.
B	Number of bootstrap replications.

Details

The parametric bootstrap method requires the assumption of the uniform censoring distribution. One must notice that such assumption is not always true in real data analysis.

Value

n	Sample size.
count	Iteration number.
random	Randomization number.
Theta	Copula parameter.
Theta.B	Copula parameter (SE and CI are calculated by parametric bootstrap method).
Alpha	Common positive scale parameter for the Pareto margin.
Alpha.B	Common positive scale parameter for the Pareto margin (SE and CI are calculated by parametric bootstrap method).
Gamma	Common positive shape parameter for the Pareto margin.
Gamma.B	Common positive shape parameter for the Pareto margin (SE and CI are calculated by parametric bootstrap method).
logL	Log-likelihood value under the fitted model.
AIC	AIC value under the fitted model.
BIC	BIC value under the fitted model.

References

Shih J-H, Lee W, Sun L-H, Emura T (2019), Fitting competing risks data to bivariate Pareto models, Communications in Statistics - Theory and Methods, 48:1193-1220.

Examples

```
t.event = c(72,40,20,65,24,46,62,61,60,60,59,59,49,20, 3,58,29,26,52,20,
            51,51,31,42,38,69,39,33, 8,13,33, 9,21,66, 5,27, 2,20,19,60,
            32,53,53,43,21,74,72,14,33, 8,10,51, 7,33, 3,43,37, 5, 6, 2,
            5,64, 1,21,16,21,12,75,74,54,73,36,59, 6,58,16,19,39,26,60,
            43, 7, 9,67,62,17,25, 0, 5,34,59,31,58,30,57, 5,55,55,52, 0,
            51,17,70,74,74,20, 2, 8,27,23, 1,52,51, 6, 0,26,65,26, 6, 6,
            68,33,67,23, 6,11, 6,57,57,29, 9,53,51, 8, 0,21,27,22,12,68,
            21,68, 0, 2,14,18, 5,60,40,51,50,46,65, 9,21,27,54,52,75,30,
            70,14, 0,42,12,40, 2,12,53,11,18,13,45, 8,28,67,67,24,64,26,
```

```
              57,32,42,20,71,54,64,51, 1, 2, 0,54,69,68,67,66,64,63,35,62,
              7,35,24,57, 1, 4,74, 0,51,36,16,32,68,17,66,65,19,41,28, 0,
              46,63,60,59,46,63, 8,74,18,33,12, 1,66,28,30,57,50,39,40,24,
              6,30,58,68,24,33,65, 2,64,19,15,10,12,53,51, 1,40,40,66, 2,
              21,35,29,54,37,10,29,71,12,13,27,66,28,31,12, 9,21,19,51,71,
              76,46,47,75,75,49,75,75,31,69,74,25,72,28,36, 8,71,60,14,22,
              67,62,68,68,27,68,68,67,67, 3,49,12,30,67, 5,65,24,66,36,66,
              40,13,40, 0,14,45,64,13,24,15,26, 5,63,35,61,61,50,57,21,26,
              11,59,42,27,50,57,57, 0, 1,54,53,23, 8,51,27,52,52,52,45,48,
              18, 2, 2,35,75,75, 9,39, 0,26,17,43,53,47,11,65,16,21,64, 7,
              38,55, 5,28,38,20,24,27,31, 9, 9,11,56,36,56,15,51,33,70,32,
              5,23,63,30,53,12,58,54,36,20,74,34,70,25,65, 4,10,58,37,56,
              6, 0,70,70,28,40,67,36,23,23,62,62,62, 2,34, 4,12,56, 1, 7,
              4,70,65, 7,30,40,13,22, 0,18,64,13,26, 1,16,33,22,30,53,53,
              7,61,40, 9,59, 7,12,46,50, 0,52,19,52,51,51,14,27,51, 5, 0,
              41,53,19)

event1 = c(0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,
           0,0,1,0,0,0,1,0,1,1,0,1,1,1,1,0,0,1,1,0,
           1,0,0,1,1,0,0,1,0,0,0,1,0,1,0,0,1,0,1,1,
           1,0,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,1,1,0,0,0,0,0,1,1,0,0,1,0,0,
           0,0,0,1,0,0,1,0,0,0,0,0,0,0,0,0,1,0,1,0,
           0,0,0,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,1,1,0,1,0,0,0,0,1,0,0,0,0,0,
           1,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,0,1,0,0,1,1,0,1,0,0,1,1,0,0,
           1,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,1,0,0,0,
           0,0,1,0,1,0,0,0,0,1,1,1,1,0,0,0,1,1,0,0,
           1,1,1,1,0,0,1,0,1,1,1,1,1,1,1,0,1,1,0,1,
           0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,
           0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,
           0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,
           1,0,0,0,0,0,0,1,0,0,0,0,1,0,1,0,1,0,0,1,
           1,1,0,1,1,1,1,1,1,1,1,0,1,1,0,0,0,0,0,0,
           0,0,0,1,0,0,0,0,1,0,0,1,0,1,0,1,1,0,1,0,
           1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,
           1,0,0,1,0,0,0,1,0,1,0,0,1,0,0,0,1,1,0,1,
           1,1,1,0,0,0,1,0,0,0,0,0,0,0,0,1,1,0,0,0,
           0,0,1)

event2 = c(0,1,1,0,0,1,0,0,0,0,0,0,0,1,1,0,1,1,0,1,
           0,0,0,1,1,0,0,1,0,0,1,0,0,0,0,1,1,0,0,0,
           0,0,0,0,0,0,0,0,1,1,1,0,1,0,1,1,0,1,0,0,
           0,0,1,0,1,1,1,0,0,0,0,1,1,1,1,1,1,1,1,1,
           1,1,1,0,1,1,1,1,1,1,0,1,0,1,0,1,0,0,0,1,
           0,1,1,0,0,1,0,0,1,1,1,0,0,0,0,1,1,0,1,1,
           0,1,0,0,1,1,0,0,0,1,1,0,0,1,1,1,0,1,0,0,
           1,0,1,0,0,1,0,0,1,0,1,1,0,1,1,1,0,0,0,1,
           0,1,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,1,0,1,
           0,0,1,1,0,1,0,1,1,1,0,1,0,0,0,0,0,0,1,0,
```

```
          1,1,1,0,1,1,1,0,1,1,0,0,0,0,0,0,0,0,1,1,
          0,0,0,0,1,0,1,0,1,1,1,1,0,1,1,1,0,1,1,1,
          1,1,0,0,0,1,0,1,0,0,0,0,0,0,0,1,0,0,0,1,
          0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,
          0,0,1,0,0,1,0,0,1,0,0,1,0,1,1,0,0,1,1,1,
          1,1,0,0,1,0,0,0,0,1,1,1,1,0,1,1,1,0,1,0,
          1,1,1,1,1,1,0,1,1,1,1,0,0,1,0,0,1,1,1,0,
          1,0,0,1,1,0,0,1,1,0,0,1,1,1,1,0,0,0,1,1,
          0,1,1,1,0,0,1,0,1,1,1,1,0,1,0,0,0,1,0,0,
          0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,1,0,1,
          1,1,0,0,1,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,
          0,1,0,0,1,1,0,1,1,1,0,0,0,1,0,1,0,0,1,1,
          0,0,0,0,1,1,1,0,1,0,1,1,0,1,1,1,0,0,1,0,
          0,0,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0,0,1,1,
          1,0,0)

library(Bivariate.Pareto)
set.seed(10)
MLE.Frank.Pareto.com(t.event,event1,event2,bootstrap = FALSE)
```

`MLE.SN.Pareto` *Maximum likelihood estimation for bivariate dependent competing risks data under the SNBP distribution*

Description

Maximum likelihood estimation for bivariate dependent competing risks data under the SNBP distribution (Sankaran and Nair, 1993).

Usage

```
MLE.SN.Pareto(
  t.event,
  event1,
  event2,
  Alpha0,
  Alpha1.0 = 1,
  Alpha2.0 = 1,
  Gamma.0 = 1,
  epsilon = 1e-05,
  d = exp(10),
  r.1 = 6,
  r.2 = 6,
  r.3 = 6
)
```

Arguments

t.event	Vector of the observed failure times.
event1	Vector of the indicators for the failure cause 1.
event2	Vector of the indicators for the failure cause 2.
Alpha0	Copula parameter α_0 with restricted range.
Alpha1.0	Initial guess for the scale parameter α_1 with default value 1.
Alpha2.0	Initial guess for the scale parameter α_2 with default value 1.
Gamma.0	Initial guess for the common shape parameter γ with default value 1.
epsilon	Positive tunning parameter in the NR algorithm with default value 10^{-5}.
d	Positive tunning parameter in the NR algorithm with default value e^{10}.
r.1	Positive tunning parameter in the NR algorithm with default value 1.
r.2	Positive tunning parameter in the NR algorithm with default value 1.
r.3	Positive tunning parameter in the NR algorithm with default value 1.

Details

The admissible range of Alpha0 (α_0) is $0 \leq \alpha_0 \leq (\gamma+1)\alpha_1\alpha_2$.

To adapt our functions to dependent censoring models in Emura and Chen (2018), one can simply set event2 = 1-event1.

Value

n	Sample size.
count	Iteration number.
random	Randomization number.
Alpha1	Positive scale parameter for the Pareto margin (failure cause 1).
Alpha2	Positive scale parameter for the Pareto margin (failure cause 2).
Gamma	Common positive shape parameter for the Pareto margins.
MedX	Median lifetime due to failure cause 1.
MedY	Median lifetime due to failure cause 2.
MeanX	Mean lifetime due to failure cause 1.
MeanY	Mean lifetime due to failure cause 2.
logL	Log-likelihood value under the fitted model.
AIC	AIC value under the fitted model.
BIC	BIC value under the fitted model.

References

Sankaran PG, Nair NU (1993), A bivariate Pareto model and its applications to reliability, Naval Research Logistics, 40(7): 1013-1020.

Emura T, Chen Y-H (2018) Analysis of Survival Data with Dependent Censoring, Copula-Based Approaches, JSS Research Series in Statistics, Springer, Singapore.

Shih J-H, Lee W, Sun L-H, Emura T (2019), Fitting competing risks data to bivariate Pareto models, Communications in Statistics - Theory and Methods, 48:1193-1220.

Examples

```
t.event = c(72,40,20,65,24,46,62,61,60,60,59,59,49,20, 3,58,29,26,52,20,
            51,51,31,42,38,69,39,33, 8,13,33, 9,21,66, 5,27, 2,20,19,60,
            32,53,53,43,21,74,72,14,33, 8,10,51, 7,33, 3,43,37, 5, 6, 2,
            5,64, 1,21,16,21,12,75,74,54,73,36,59, 6,58,16,19,39,26,60,
            43, 7, 9,67,62,17,25, 0, 5,34,59,31,58,30,57, 5,55,55,52, 0,
            51,17,70,74,74,20, 2, 8,27,23, 1,52,51, 6, 0,26,65,26, 6, 6,
            68,33,67,23, 6,11, 6,57,57,29, 9,53,51, 8, 0,21,27,22,12,68,
            21,68, 0, 2,14,18, 5,60,40,51,50,46,65, 9,21,27,54,52,75,30,
            70,14, 0,42,12,40, 2,12,53,11,18,13,45, 8,28,67,67,24,64,26,
            57,32,42,20,71,54,64,51, 1, 2, 0,54,69,68,67,66,64,63,35,62,
            7,35,24,57, 1, 4,74, 0,51,36,16,32,68,17,66,65,19,41,28, 0,
            46,63,60,59,46,63, 8,74,18,33,12, 1,66,28,30,57,50,39,40,24,
            6,30,58,68,24,33,65, 2,64,19,15,10,12,53,51, 1,40,40,66, 2,
            21,35,29,54,37,10,29,71,12,13,27,66,28,31,12, 9,21,19,51,71,
            76,46,47,75,75,49,75,75,31,69,74,25,72,28,36, 8,71,60,14,22,
            67,62,68,68,27,68,68,67,67, 3,49,12,30,67, 5,65,24,66,36,66,
            40,13,40, 0,14,45,64,13,24,15,26, 5,63,35,61,61,50,57,21,26,
            11,59,42,27,50,57,57, 0, 1,54,53,23, 8,51,27,52,52,52,45,48,
            18, 2, 2,35,75,75, 9,39, 0,26,17,43,53,47,11,65,16,21,64, 7,
            38,55, 5,28,38,20,24,27,31, 9, 9,11,56,36,56,15,51,33,70,32,
            5,23,63,30,53,12,58,54,36,20,74,34,70,25,65, 4,10,58,37,56,
            6, 0,70,70,28,40,67,36,23,23,62,62,62, 2,34, 4,12,56, 1, 7,
            4,70,65, 7,30,40,13,22, 0,18,64,13,26, 1,16,33,22,30,53,53,
            7,61,40, 9,59, 7,12,46,50, 0,52,19,52,51,51,14,27,51, 5, 0,
            41,53,19)

event1 = c(0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,
           0,0,1,0,0,0,1,0,1,1,0,1,1,1,1,0,0,1,1,0,
           1,0,0,1,1,0,0,1,0,0,0,1,0,1,0,0,1,0,1,1,
           1,0,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,1,1,0,0,0,0,0,1,1,0,0,1,0,0,
           0,0,0,1,0,0,1,0,0,0,0,0,0,0,0,0,1,0,1,0,
           0,0,0,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,1,1,0,1,0,0,0,0,1,0,0,0,0,0,
           1,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,0,1,0,0,1,1,0,1,0,0,1,1,0,0,
           1,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,1,0,0,0,
           0,0,1,0,1,0,0,0,0,1,1,1,1,0,0,0,1,1,0,0,
           1,1,1,1,0,0,1,0,1,1,1,1,1,1,1,0,1,1,0,1,
           0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,
           0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
           0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,
           0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,
           1,0,0,0,0,0,0,1,0,0,0,0,1,0,1,0,1,0,0,1,
           1,1,0,1,1,1,1,1,1,1,1,0,1,1,0,0,0,0,0,0,
           0,0,0,1,0,0,0,0,1,0,0,1,0,1,0,1,1,0,1,0,
           1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,
           1,0,0,1,0,0,0,1,0,1,0,0,1,0,0,0,1,1,0,1,
           1,1,1,0,0,0,1,0,0,0,0,0,0,0,0,1,1,0,0,0,
           0,0,1)
```

```
event2 = c(0,1,1,0,0,1,0,0,0,0,0,0,0,1,1,0,1,1,0,1,
           0,0,0,1,1,0,0,1,0,0,1,0,0,0,0,1,1,0,0,0,
           0,0,0,0,0,0,0,0,1,1,1,0,1,0,1,1,0,1,0,0,
           0,0,1,0,1,1,1,0,0,0,0,1,1,1,1,1,1,1,1,1,
           1,1,1,0,1,1,1,1,1,1,0,1,0,1,0,1,0,0,0,1,
           0,1,1,0,0,1,0,0,1,1,1,0,0,0,0,1,1,0,1,1,
           0,1,0,0,1,1,0,0,0,1,1,0,0,1,1,1,0,1,0,0,
           1,0,1,0,0,1,0,0,1,0,1,1,0,1,1,1,0,0,0,1,
           0,1,1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,1,0,1,
           0,0,1,1,0,1,0,1,1,1,0,1,0,0,0,0,0,0,1,0,
           1,1,1,0,1,1,1,0,1,1,0,0,0,0,0,0,0,0,1,1,
           0,0,0,0,1,0,1,0,1,1,1,1,0,1,1,1,0,1,1,1,
           1,1,0,0,0,1,0,1,0,0,0,0,0,0,0,1,0,0,0,1,
           0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,
           0,0,1,0,0,1,0,0,1,0,0,1,0,1,1,0,0,1,1,1,
           1,1,0,0,1,0,0,0,0,1,1,1,1,0,1,1,1,0,1,0,
           1,1,1,1,1,1,0,1,1,1,1,0,0,1,0,0,1,1,1,0,
           1,0,0,1,1,0,0,1,1,0,0,1,1,1,1,0,0,0,1,1,
           0,1,1,1,0,0,1,0,1,1,1,1,0,1,0,0,0,1,0,0,
           0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,1,0,1,
           1,1,0,0,1,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,
           0,1,0,0,1,1,0,1,1,1,0,0,0,1,0,1,0,0,1,1,
           0,0,0,0,1,1,1,0,1,0,1,1,0,1,1,1,0,0,1,0,
           0,0,0,1,0,1,0,1,0,1,0,1,0,0,0,0,0,0,1,1,
           1,0,0)

library(Bivariate.Pareto)
set.seed(10)
MLE.SN.Pareto(t.event,event1,event2,Alpha0 = 7e-5)
```

`SN.Pareto` *Generate samples from the SNBP distribution*

Description

Generate samples from the Sankaran and Nair bivairate Pareto (SNBP) distribution (Sankaran and Nair, 1993).

Usage

```
SN.Pareto(n, Alpha0, Alpha1, Alpha2, Gamma)
```

Arguments

`n`	Sample size.
`Alpha0`	Copula parameter α_0 with restricted range.
`Alpha1`	Positive scale parameter α_1 for the Pareto margin.
`Alpha2`	Positive scale parameter α_2 for the Pareto margin.
`Gamma`	Common positive shape parameter γ for the Pareto margins.

Details

The admissible range of Alpha0 (α_0) is $0 \le \alpha_0 \le (\gamma + 1)\alpha_1\alpha_2$.

Value

X	X is asscoiated with the parameters Alpha1 and Gamma.
Y	Y is asscoiated with the parameters Alpha2 and Gamma.

References

Sankaran PG, Nair NU (1993), A bivariate Pareto model and its applications to reliability, Naval Research Logistics, 40(7): 1013-1020.

Shih J-H, Lee W, Sun L-H, Emura T (2019), Fitting competing risks data to bivariate Pareto models, Communications in Statistics - Theory and Methods, 48:1193-1220.

Examples

```
library(Bivariate.Pareto)
SN.Pareto(5,2,1,1,1)
```

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