

# Package ‘BKTR’

January 20, 2025

**Version** 0.2.0

**Title** Bayesian Kernelized Tensor Regression

**Description** Facilitates scalable spatiotemporally varying coefficient modelling with Bayesian kernelized tensor regression.

The important features of this package are:

- (a) Enabling local temporal and spatial modeling of the relationship between the response variable and covariates.
- (b) Implementing the model described by Lei et al. (2023) <[doi:10.48550/arXiv.2109.00046](https://doi.org/10.48550/arXiv.2109.00046)>.
- (c) Using a Bayesian Markov Chain Monte Carlo (MCMC) algorithm to sample from the posterior distribution of the model parameters.
- (d) Employing a tensor decomposition to reduce the number of estimated parameters.
- (e) Accelerating tensor operations and enabling graphics processing unit (GPU) acceleration with the 'torch' package.

**Depends** R (>= 4.0.0)

**Encoding** UTF-8

**Imports** torch (>= 0.13.0), R6, R6P, ggplot2, ggmap, data.table

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**RoxygenNote** 7.2.3

**Collate** 'samplers.R' 'tensor\_ops.R' 'result\_logger.R'  
'likelihood\_evaluator.R' 'distances.R' 'kernels.R' 'bktr.R'  
'examples.R' 'plots.R' 'utils.R'

**Suggests** knitr, rmarkdown, R.rsp

**LazyData** true

**VignetteBuilder** knitr, rmarkdown, R.rsp

**BugReports** <https://github.com/julien-hec/BKTR/issues>

**NeedsCompilation** no

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\*.Kernel                      *Operator overloading for kernel multiplication*

---

**Description**

Operator overloading for kernel multiplication

**Usage**

```
## S3 method for class 'Kernel'  
k1 * k2
```

**Arguments**

k1                      Kernel: The left kernel to use for composition  
k2                      Kernel: The right kernel to use for composition

**Value**

A new KernelMulComposed object.

**Examples**

```
# Create a new locally periodic kernel  
k_loc_per <- KernelSE$new() * KernelPeriodic$new()  
# Set the kernel's positions  
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))  
k_loc_per$set_positions(positions_df)  
# Generate the kernel's covariance matrix  
k_loc_per$kernel_gen()
```

---

+.Kernel                      *Operator overloading for kernel addition*

---

**Description**

Operator overloading for kernel addition

**Usage**

```
## S3 method for class 'Kernel'  
k1 + k2
```

**Arguments**

k1                    Kernel: The left kernel to use for composition  
 k2                    Kernel: The right kernel to use for composition

**Value**

A new KernelAddComposed object.

**Examples**

```
# Create a new additive kernel
k_rq_plus_per <- KernelRQ$new() + KernelPeriodic$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_rq_plus_per$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_rq_plus_per$kernel_gen()
```

---

 BixiData

*BIXI Data Class*


---

**Description**

R6 class encapsulating all BIXI dataframes. It is also possible to use a light version of the dataset by using the `is_light` parameter. In this case, the dataset is reduced to its first 25 stations and first 50 days. The light version is only used for testing and short examples.

**Public fields**

departure\_df The departure dataframe  
 spatial\_features\_df The spatial features dataframe  
 temporal\_features\_df The temporal features dataframe  
 spatial\_positions\_df The spatial positions dataframe  
 temporal\_positions\_df The temporal positions dataframe  
 data\_df The data dataframe  
 is\_light Whether the light version of the dataset is used

**Methods****Public methods:**

- [BixiData\\$new\(\)](#)
- [BixiData\\$clone\(\)](#)

**Method** `new()`: Initialize the BIXI data class

*Usage:*

```
BixiData$new(is_light = FALSE)
```

*Arguments:*

`is_light` Whether the light version of the dataset is used, defaults to FALSE.

*Returns:* A new BIXI data instance

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
BixiData$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

**Examples**

```
# Create a light BIXI data collection instance containing multiple dataframes
# This only uses the first 25 stations and 50 days of the full dataset
bixi_data <- BixiData$new(is_light = TRUE)
# Dataframe containing the position (latitude and longitude) of M stations
bixi_data$spatial_positions_df
# Dataframe containing the time position of N days (0 to N-1)
bixi_data$temporal_positions_df
# Dataframe with spatial and temporal features for each day and station (M x N rows)
bixi_data$data_df
```

---

bixi\_spatial\_features *Spatial Features of Montreal BIXI Stations in 2019*

---

**Description**

These data represent 14 spatial features (columns) for 587 bike sharing stations (rows) located at different geographical coordinates (longitude, latitude) in Montreal. The Montreal based bike sharing company is named BIXI. The first column contains the descriptive label affected to each station and the other columns contain information about the infrastructure, points of interests, walkscore and population surrounding each station for 2019.

**Usage**

```
data("bixi_spatial_features")
```

**Format**

A data frame with 587 observations on the following 14 variables.

`location` a character vector

`area_park` a numeric vector

len\_cycle\_path a numeric vector  
len\_major\_road a numeric vector  
len\_minor\_road a numeric vector  
num\_metro\_stations a numeric vector  
num\_other\_commercial a numeric vector  
num\_restaurants a numeric vector  
num\_university a numeric vector  
num\_pop a numeric vector  
num\_bus\_stations a numeric vector  
num\_bus\_routes a numeric vector  
walkscore a numeric vector  
capacity a numeric vector

### Source

Wang, X., Cheng, Z., Trépanier, M., & Sun, L. (2021). Modeling bike-sharing demand using a regression model with spatially varying coefficients. *Journal of Transport Geography*, 93, 103059.

### References

Reference for the BIXI station informations: BIXI Montréal (2023). "Open Data." Accessed: 2023-07-11, URL <https://bixi.com/en/open-data>.

Reference for point of interests and infrastructure informations: DMTI Spatial Inc (2019). "Enhanced Point of Interest (DMTI)." URL <https://www.dmtispacial.com>.

Reference for Walkscore: Walk Score (2023). "Walk Score Methodology." Accessed: 2023-07-11, URL <https://www.walkscore.com/methodology.shtml>.

The population information comes from the 2016 Canada census data at a dissemination block level.

---

bixi\_spatial\_locations

*Spatial Locations of Montreal BIXI Stations in 2019*

---

### Description

Data points representing the spatial locations of 587 bike sharing stations for the Montreal based bike sharing company named BIXI. The dataframe contains a label to identify each station and its associated longitude and latitude coordinates.

### Usage

```
data("bixi_spatial_locations")
```

**Format**

A data frame with 587 observations on the following 3 variables.

location a character vector

latitude a numeric vector

longitude a numeric vector

**Source**

Wang, X., Cheng, Z., Trépanier, M., & Sun, L. (2021). Modeling bike-sharing demand using a regression model with spatially varying coefficients. *Journal of Transport Geography*, 93, 103059.

**References**

BIXI Montréal (2023). "Open Data." Accessed: 2023-07-11, URL <https://bixi.com/en/open-data>.

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bixi\_station\_departures

*Daily Departure from BIXI Stations in 2019*

---

**Description**

These data capture the number of daily departure for 587 bike sharing stations (rows) through 197 days (columns). The data is limited to the 2019 season of a Montreal based bike sharing company named BIXI.

**Usage**

```
data("bixi_station_departures")
```

**Format**

A data frame with 587 rows and 197 columns.

**Source**

Wang, X., Cheng, Z., Trépanier, M., & Sun, L. (2021). Modeling bike-sharing demand using a regression model with spatially varying coefficients. *Journal of Transport Geography*, 93, 103059.

**References**

BIXI Montréal (2023). "Open Data." Accessed: 2023-07-11, URL <https://bixi.com/en/open-data>.

---

bixi\_temporal\_features

*Temporal Features in Montreal applicable to BIXI for 2019*

---

### Description

These data represent the temporal features in Montreal applicable to a Montreal based bike sharing company named BIXI. The data include six features (columns) for 196 days (rows). The time column represent the label associated to each captured time for the 2019 season of BIXI. The other columns contain information about Montreal weather and applicable holidays for each day.

### Usage

```
data("bixi_temporal_features")
```

### Format

A data frame with 196 observations on the following 6 variables.

time a IDate

humidity a numeric vector

max\_temp\_f a numeric vector

mean\_temp\_c a numeric vector

total\_precip\_mm a numeric vector

holiday a numeric vector

### Source

Lei, M., Labbe, A., & Sun, L. (2021). Scalable Spatiotemporally Varying Coefficient Modelling with Bayesian Kernelized Tensor Regression. arXiv preprint arXiv:2109.00046.

### References

The weather data is sourced from the Environment and Climate Change Canada Historical Climate Data website.

The holiday column is specifying if a date is a holiday or not, according to the Quebec government.

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 bixi\_temporal\_locations

*Temporal indices for the 2019 BIXI season*


---

### Description

These data represent 196 temporal indices (rows) related to each day of the 2019 season of Montreal based bike sharing company named BIXI. The time column represent the label associated to each day and the time\_index column represent the location in time space of each day when compared to each other. Since no days are missing and they are all spaced by exactly one day, the time\_index is simply a range from 0 to 195.

### Usage

```
data("bixi_temporal_locations")
```

### Format

A data frame with 196 observations on the following 2 variables.

time a IDate

time\_index a numeric vector

### Source

Lei, M., Labbe, A., & Sun, L. (2021). Scalable Spatiotemporally Varying Coefficient Modelling with Bayesian Kernelized Tensor Regression. arXiv preprint arXiv:2109.00046.

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 BKTRRegressor

*R6 class encapsulating the BKTR regression elements*


---

### Description

A BKTRRegressor holds all the key elements to accomplish the MCMC sampling algorithm (**Algorithm 1** of the paper).

### Public fields

data\_df The dataframe containing all the covariates through time and space (including the response variable)

y The response variable tensor

omega The tensor indicating which response values are not missing

covariates The tensor containing all the covariates

covariates\_dim The dimensions of the covariates tensor

logged\_params\_tensor The tensor containing all the sampled hyperparameters  
 tau The precision hyperparameter  
 spatial\_decomp The spatial covariate decomposition  
 temporal\_decomp The temporal covariate decomposition  
 covs\_decomp The feature covariate decomposition  
 result\_logger The result logger instance used to store the results of the MCMC sampling  
 has\_completed\_sampling Boolean showing wheter the MCMC sampling has been completed  
 spatial\_kernel The spatial kernel used  
 temporal\_kernel The temporal kernel used  
 spatial\_positions\_df The dataframe containing the spatial positions  
 temporal\_positions\_df The dataframe containing the temporal positions  
 spatial\_params\_sampler The spatial kernel hyperparameter sampler  
 temporal\_params\_sampler The temporal kernel hyperparameter sampler  
 tau\_sampler The tau hyperparameter sampler  
 precision\_matrix\_sampler The precision matrix sampler  
 spatial\_ll\_evaluator The spatial likelihood evaluator  
 temporal\_ll\_evaluator The temporal likelihood evaluator  
 rank\_decomp The rank of the CP decomposition  
 burn\_in\_iter The number of burn in iterations  
 sampling\_iter The number of sampling iterations  
 max\_iter The total number of iterations  
 a\_0 The initial value for the shape in the gamma function generating tau  
 b\_0 The initial value for the rate in the gamma function generating tau  
 formula The formula used to specify the relation between the response variable and the covariates  
 spatial\_labels The spatial labels  
 temporal\_labels The temporal labels  
 feature\_labels The feature labels  
 geo\_coords\_projector The geographic coordinates projector

### Active bindings

summary A summary of the BKTRRegressor instance  
 beta\_covariates\_summary A dataframe containing the summary of the beta covariates  
 y\_estimates A dataframe containing the y estimates  
 imputed\_y\_estimates A dataframe containing the imputed y estimates  
 beta\_estimates A dataframe containing the beta estimates  
 hyperparameters\_per\_iter\_df A dataframe containing the beta estimates per iteration  
 decomposition\_tensors List of all used decomposition tensors

## Methods

### Public methods:

- `BKTRRegressor$new()`
- `BKTRRegressor$mcmc_sampling()`
- `BKTRRegressor$predict()`
- `BKTRRegressor$get_iterations_betas()`
- `BKTRRegressor$get_beta_summary_df()`
- `BKTRRegressor$clone()`

**Method** `new()`: Create a new BKTRRegressor object.

*Usage:*

```
BKTRRegressor$new(
  data_df,
  spatial_positions_df,
  temporal_positions_df,
  rank_decomp = 10,
  burn_in_iter = 500,
  sampling_iter = 500,
  formula = NULL,
  spatial_kernel = KernelMatern$new(smoothness_factor = 3),
  temporal_kernel = KernelSE$new(),
  sigma_r = 0.01,
  a_0 = 1e-06,
  b_0 = 1e-06,
  has_geo_coords = TRUE,
  geo_coords_scale = 10
)
```

*Arguments:*

`data_df` `data.table`: A dataframe containing all the covariates through time and space. It is important that the dataframe has a two indexes named 'location' and 'time' respectively. The dataframe should also contain every possible combinations of 'location' and 'time' (i.e. even missing rows should be filled present but filled with NaN). So if the dataframe has 10 locations and 5 time points, it should have 50 rows (10 x 5). If formula is None, the dataframe should contain the response variable 'Y' as the first column. Note that the covariate columns cannot contain NaN values, but the response variable can.

`spatial_positions_df` `data.table`: Spatial kernel input tensor used to calculate covariates' distance. Vector of length equal to the number of location points.

`temporal_positions_df` `data.table`: Temporal kernel input tensor used to calculate covariate distance. Vector of length equal to the number of time points.

`rank_decomp` `Integer`: Rank of the CP decomposition (Paper –  $R$ ). Defaults to 10.

`burn_in_iter` `Integer`: Number of iteration before sampling (Paper –  $K_1$ ). Defaults to 500.

`sampling_iter` `Integer`: Number of sampling iterations (Paper –  $K_2$ ). Defaults to 500.

`formula` A Wilkinson R formula to specify the relation between the response variable 'Y' and the covariates. If Null, the first column of the data frame will be used as the response variable and all the other columns will be used as the covariates. Defaults to Null.

**spatial\_kernel** Kernel: Spatial kernel Used. Defaults to a KernelMatern(smoothness\_factor=3).  
**temporal\_kernel** Kernel: Temporal kernel used. Defaults to KernelSE().  
**sigma\_r** Numeric: Variance of the white noise process ( $\tau^{-1}$ ) defaults to 1E-2.  
**a\_0** Numeric: Initial value for the shape ( $\alpha$ ) in the gamma function generating tau defaults to 1E-6.  
**b\_0** Numeric: Initial value for the rate ( $\beta$ ) in the gamma function generating tau defaults to 1E-6.  
**has\_geo\_coords** Boolean: Whether the spatial positions df use geographic coordinates (latitude, longitude). Defaults to TRUE.  
**geo\_coords\_scale** Numeric: Scale factor to convert geographic coordinates to euclidean 2D space via Mercator projection using x & y domains of [-scale/2, +scale/2]. Only used if has\_geo\_coords is TRUE. Defaults to 10.

*Returns:* A new BKTRRegressor object.

**Method** `mcmc_sampling()`: Launch the MCMC sampling process.

For a predefined number of iterations:

1. Sample spatial kernel hyperparameters
2. Sample temporal kernel hyperparameters
3. Sample the precision matrix from a wishart distribution
4. Sample a new spatial covariate decomposition
5. Sample a new feature covariate decomposition
6. Sample a new temporal covariate decomposition
7. Calculate respective errors for the iterations
8. Sample a new tau value
9. Collect all the important data for the iteration

*Usage:*

```
BKTRRegressor$mcmc_sampling()
```

*Returns:* NULL Results are stored and can be accessed via `summary()`

**Method** `predict()`: Use interpolation to predict betas and response values for new data.

*Usage:*

```

BKTRRegressor$predict(
  new_data_df,
  new_spatial_positions_df = NULL,
  new_temporal_positions_df = NULL,
  jitter = 1e-05
)

```

*Arguments:*

**new\_data\_df** data.table: New covariates. Must have the same columns as the covariates used to fit the model. The index should contain the combination of all old spatial coordinates with all new temporal coordinates, the combination of all new spatial coordinates with all old temporal coordinates, and the combination of all new spatial coordinates with all new temporal coordinates.

`new_spatial_positions_df` data.table or NULL: A data frame containing the new spatial positions. Defaults to NULL.

`new_temporal_positions_df` data.table or NULL: A data frame containing the new temporal positions. Defaults to NULL.

`jitter` Numeric or NULL: A small value to add to the diagonal of the precision matrix. Defaults to NULL.

*Returns:* List: A list of two dataframes. The first represents the beta forecasted for all new spatial locations or temporal points. The second represents the forecasted response for all new spatial locations or temporal points.

**Method** `get_iterations_betas()`: Return all sampled betas through sampling iterations for a given set of spatial, temporal and feature labels. Useful for plotting the distribution of sampled beta values.

*Usage:*

```
BKTRRegressor$get_iterations_betas(
  spatial_label,
  temporal_label,
  feature_label
)
```

*Arguments:*

`spatial_label` String: The spatial label for which we want to get the betas

`temporal_label` String: The temporal label for which we want to get the betas

`feature_label` String: The feature label for which we want to get the betas

*Returns:* A list containing the sampled betas through iteration for the given labels

**Method** `get_beta_summary_df()`: Get a summary of estimated beta values. If no labels are given, then the summary is for all the betas. If labels are given, then the summary is for the given labels.

*Usage:*

```
BKTRRegressor$get_beta_summary_df(
  spatial_labels = NULL,
  temporal_labels = NULL,
  feature_labels = NULL
)
```

*Arguments:*

`spatial_labels` vector: The spatial labels used in summary. If NULL, then all spatial labels are used. Defaults to NULL.

`temporal_labels` vector: The temporal labels used in summary. If NULL, then all temporal labels are used. Defaults to NULL.

`feature_labels` vector: The feature labels used in summary. If NULL, then all feature labels are used. Defaults to NULL.

*Returns:* A new data.table with the beta summary for the given labels.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
BKTRRegressor$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

## Examples

```
# Create a BIXI data collection instance containing multiple dataframes
bixi_data <- BixiData$new(is_light = TRUE) # Use light version for example

# Create a BKTRRegressor instance
bktr_regressor <- BKTRRegressor$new(
  formula = nb_departure ~ 1 + mean_temp_c + area_park,
  data_df <- bixi_data$data_df,
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)

# Launch the MCMC sampling
bktr_regressor$mcmc_sampling()

# Get the summary of the bktr regressor
summary(bktr_regressor)

# Get estimated response variables for missing values
bktr_regressor$imputed_y_estimates

# Get the list of sampled betas for given spatial, temporal and feature labels
bktr_regressor$get_iterations_betas(
  spatial_label = bixi_data$spatial_positions_df$location[1],
  temporal_label = bixi_data$temporal_positions_df$time[1],
  feature_label = 'mean_temp_c')

# Get the summary of all betas for the 'mean_temp_c' feature
bktr_regressor$get_beta_summary_df(feature_labels = 'mean_temp_c')

## PREDICTION EXAMPLE ##
# Create a light version of the BIXI data collection instance
bixi_data <- BixiData$new(is_light = TRUE)
# Simplify variable names
data_df <- bixi_data$data_df
spa_pos_df <- bixi_data$spatial_positions_df
temp_pos_df <- bixi_data$temporal_positions_df

# Keep some data aside for prediction
new_spa_pos_df <- spa_pos_df[1:2, ]
new_temp_pos_df <- temp_pos_df[1:5, ]
reg_spa_pos_df <- spa_pos_df[-(1:2), ]
reg_temp_pos_df <- temp_pos_df[-(1:5), ]
reg_data_df_mask <- data_df$location %in% reg_spa_pos_df$location &
  data_df$time %in% reg_temp_pos_df$time
reg_data_df <- data_df[reg_data_df_mask, ]
```

```

new_data_df <- data_df[!reg_data_df_mask, ]

# Launch mcmc sampling on regression data
bktr_regressor <- BKTRRegressor$new(
  formula = nb_departure ~ 1 + mean_temp_c + area_park,
  data_df = reg_data_df,
  spatial_positions_df = reg_spa_pos_df,
  temporal_positions_df = reg_temp_pos_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Predict response values for new data
bktr_regressor$predict(
  new_data_df = new_data_df,
  new_spatial_positions_df = new_spa_pos_df,
  new_temporal_positions_df = new_temp_pos_df)

```

---

CompositionOps

*Kernel Composition Operations*


---

### Description

Kernel Composition Operations Enum. Possibilities of operation between two kernels to generate a new composed kernel. The values are: MUL and ADD.

### Usage

CompositionOps

### Format

An object of class list of length 2.

---

Kernel

*Base R6 class for Kernels*


---

### Description

Abstract base class for kernels (Should not be instantiated)

**Public fields**

`kernel_variance` The variance of the kernel  
`jitter_value` The jitter value to add to the kernel matrix  
`distance_matrix` The distance matrix between points in a tensor format  
`name` The kernel's name  
`parameters` The parameters of the kernel (list of `KernelParameter`)  
`covariance_matrix` The covariance matrix of the kernel in a tensor format  
`positions_df` The positions of the points in a dataframe format  
`has_dist_matrix` Identify if the kernel has a distance matrix or not

**Methods****Public methods:**

- `Kernel$new()`
- `Kernel$core_kernel_fn()`
- `Kernel$add_jitter_to_kernel()`
- `Kernel$kernel_gen()`
- `Kernel$set_positions()`
- `Kernel$plot()`
- `Kernel$clone()`

**Method** `new()`: Kernel abstract base constructor

*Usage:*

`Kernel$new(kernel_variance, jitter_value)`

*Arguments:*

`kernel_variance` Numeric: The variance of the kernel

`jitter_value` Numeric: The jitter value to add to the kernel matrix

*Returns:* A new Kernel object.

**Method** `core_kernel_fn()`: Abstract method to compute the core kernel's covariance matrix

*Usage:*

`Kernel$core_kernel_fn()`

**Method** `add_jitter_to_kernel()`: Method to add jitter to the kernel's covariance matrix

*Usage:*

`Kernel$add_jitter_to_kernel()`

**Method** `kernel_gen()`: Method to compute the kernel's covariance matrix

*Usage:*

`Kernel$kernel_gen()`

**Method** `set_positions()`: Method to set the kernel's positions and compute the distance matrix

*Usage:*

```
Kernel$set_positions(positions_df)
```

*Arguments:*

positions\_df Dataframe: The positions of the points in a dataframe format

**Method** plot(): Method to plot the kernel's covariance matrix

*Usage:*

```
Kernel$plot(show_figure = TRUE)
```

*Arguments:*

show\_figure Boolean: If TRUE, the figure is shown, otherwise it is returned

*Returns:* If show\_figure is TRUE, the figure is shown, otherwise it is returned

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

```
Kernel$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

---

 KernelAddComposed

*R6 class for Kernels Composed via Addition*


---

**Description**

R6 class automatically generated when adding two kernels together.

**Super classes**

[BKTR::Kernel](#) -> [BKTR::KernelComposed](#) -> KernelAddComposed

**Methods****Public methods:**

- [KernelAddComposed\\$new\(\)](#)
- [KernelAddComposed\\$clone\(\)](#)

**Method** new(): Create a new KernelAddComposed object.

*Usage:*

```
KernelAddComposed$new(left_kernel, right_kernel, new_name)
```

*Arguments:*

left\_kernel Kernel: The left kernel to use for composition

right\_kernel Kernel: The right kernel to use for composition

new\_name String: The name of the composed kernel

*Returns:* A new KernelAddComposed object.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

```
KernelAddComposed$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

### Examples

```
# Create a new additive kernel
k_rq_plus_per <- KernelAddComposed$new(
  left_kernel = KernelRQ$new(),
  right_kernel = KernelPeriodic$new(),
  new_name = 'SE + Periodic Kernel'
)
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_rq_plus_per$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_rq_plus_per$kernel_gen()
```

---

KernelComposed

*R6 class for Composed Kernels*

---

### Description

R6 class for Composed Kernels

### Super class

[BKTR::Kernel](#) -> KernelComposed

### Public fields

name The kernel's name

parameters The parameters of the kernel (list of KernelParameter)

left\_kernel The left kernel to use for composition

right\_kernel The right kernel to use for composition

composition\_operation The operation to use for composition

has\_dist\_matrix Identify if the kernel has a distance matrix or not

## Methods

### Public methods:

- `KernelComposed$new()`
- `KernelComposed$core_kernel_fn()`
- `KernelComposed$set_positions()`
- `KernelComposed$clone()`

**Method** `new()`: Create a new `KernelComposed` object.

*Usage:*

```
KernelComposed$new(left_kernel, right_kernel, new_name, composition_operation)
```

*Arguments:*

`left_kernel` Kernel: The left kernel to use for composition

`right_kernel` Kernel: The right kernel to use for composition

`new_name` String: The name of the composed kernel

`composition_operation` `CompositionOps`: The operation to use for composition

**Method** `core_kernel_fn()`: Method to compute the core kernel's covariance matrix

*Usage:*

```
KernelComposed$core_kernel_fn()
```

*Returns:* The core kernel's covariance matrix

**Method** `set_positions()`: Method to set the kernel's positions and compute the distance matrix

*Usage:*

```
KernelComposed$set_positions(positions_df)
```

*Arguments:*

`positions_df` `Dataframe`: The positions of the points in a dataframe format

*Returns:* `NULL`, set the kernel's positions and compute the distance matrix

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
KernelComposed$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## Examples

```
# Create a new locally periodic kernel
k_loc_per <- KernelComposed$new(
  left_kernel = KernelSE$new(),
  right_kernel = KernelPeriodic$new(),
  new_name = 'Locally Periodic Kernel',
  composition_operation = CompositionOps$MUL
)
```

```
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_loc_per$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_loc_per$kernel_gen()
```

---

KernelMatern

*R6 class for Matern Kernels*


---

### Description

R6 class for Matern Kernels

### Super class

[BKTR::Kernel](#) -> KernelMatern

### Public fields

`lengthscale` The lengthscale parameter instance of the kernel

`smoothness_factor` The smoothness factor of the kernel

`has_dist_matrix` Identify if the kernel has a distance matrix or not

### Methods

#### Public methods:

- [KernelMatern\\$new\(\)](#)
- [KernelMatern\\$get\\_smoothness\\_kernel\\_fn\(\)](#)
- [KernelMatern\\$core\\_kernel\\_fn\(\)](#)
- [KernelMatern\\$clone\(\)](#)

**Method** `new()`: Create a new KernelMatern object.

*Usage:*

```
KernelMatern$new(
  smoothness_factor = 5,
  lengthscale = KernelParameter$new(2),
  kernel_variance = 1,
  jitter_value = NULL
)
```

*Arguments:*

`smoothness_factor` Numeric: The smoothness factor of the kernel (1, 3 or 5)

`lengthscale` KernelParameter: The lengthscale parameter instance of the kernel

`kernel_variance` Numeric: The variance of the kernel

`jitter_value` Numeric: The jitter value to add to the kernel matrix

**Method** `get_smoothness_kernel_fn()`: Method to the get the smoothness kernel function for a given integer smoothness factor

*Usage:*

```
KernelMatern$get_smoothness_kernel_fn()
```

*Returns:* The smoothness kernel function

**Method** `core_kernel_fn()`: Method to compute the core kernel's covariance matrix

*Usage:*

```
KernelMatern$core_kernel_fn()
```

*Returns:* The core kernel's covariance matrix

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
KernelMatern$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## Examples

```
# Create a new Matern 3/2 kernel
k_matern <- KernelMatern$new(smoothness_factor = 3)
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_matern$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_matern$kernel_gen()
```

---

KernelMulComposed      *R6 class for Kernels Composed via Multiplication*

---

## Description

R6 class automatically generated when multiplying two kernels together.

## Super classes

[BKTR::Kernel](#) -> [BKTR::KernelComposed](#) -> KernelMulComposed

## Methods

### Public methods:

- [KernelMulComposed\\$new\(\)](#)
- [KernelMulComposed\\$clone\(\)](#)

**Method** `new()`: Create a new `KernelMulComposed` object.

*Usage:*

```
KernelMulComposed$new(left_kernel, right_kernel, new_name)
```

*Arguments:*

`left_kernel` Kernel: The left kernel to use for composition

`right_kernel` Kernel: The right kernel to use for composition

`new_name` String: The name of the composed kernel

*Returns:* A new `KernelMulComposed` object.

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
KernelMulComposed$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## Examples

```
# Create a new locally periodic kernel
k_loc_per <- KernelMulComposed$new(
  left_kernel = KernelSE$new(),
  right_kernel = KernelPeriodic$new(),
  new_name = 'Locally Periodic Kernel'
)
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_loc_per$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_loc_per$kernel_gen()
```

---

KernelParameter

*R6 class for kernel's hyperparameter*

---

## Description

`KernelParameter` contains all information and behaviour related to a kernel parameters.

**Public fields**

value The hyperparameter mean's prior value or its constant value  
 is\_fixed Says if the kernel parameter is fixed or not (if fixed, there is no sampling)  
 lower\_bound The hyperparameter's minimal value during sampling  
 upper\_bound The hyperparameter's maximal value during sampling  
 slice\_sampling\_scale The sampling range's amplitude  
 hparam\_precision Precision of the hyperparameter  
 kernel The kernel associated with the parameter (it is set at kernel instantiation)  
 name The kernel parameter's name

**Active bindings**

full\_name The kernel parameter's full name

**Methods****Public methods:**

- [KernelParameter\\$new\(\)](#)
- [KernelParameter\\$set\\_kernel\(\)](#)
- [KernelParameter\\$clone\(\)](#)

**Method** new(): Create a new KernelParameter object.

*Usage:*

```
KernelParameter$new(
  value,
  is_fixed = FALSE,
  lower_bound = DEFAULT_LBOUND,
  upper_bound = DEFAULT_UBOUND,
  slice_sampling_scale = log(10),
  hparam_precision = 1
)
```

*Arguments:*

value Numeric: The hyperparameter mean's prior value (Paper -  $\phi$ ) or its constant value  
 is\_fixed Boolean: Says if the kernel parameter is fixed or not (if fixed, there is no sampling)  
 lower\_bound Numeric: Hyperparameter's minimal value during sampling (Paper -  $\phi_{min}$ )  
 upper\_bound Numeric: Hyperparameter's maximal value during sampling (Paper -  $\phi_{max}$ )  
 slice\_sampling\_scale Numeric: The sampling range's amplitude (Paper -  $\rho$ )  
 hparam\_precision Numeric: The hyperparameter's precision

*Returns:* A new KernelParameter object.

**Method** set\_kernel(): Set Kernel for a given KernelParameter object.

*Usage:*

```
KernelParameter$set_kernel(kernel, param_name)
```

*Arguments:*

kernel Kernel: The kernel to associate with the parameter

param\_name String: The parameter's name

*Returns:* NULL, set a new kernel for the parameter

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

```
KernelParameter$clone(deep = FALSE)
```

*Arguments:*

deep Whether to make a deep clone.

**Examples**

```
# A kernel parameter can be a constant value
const_param <- KernelParameter$new(7, is_fixed = TRUE)
# It can otherwise be sampled and have its value updated through sampling
samp_param <- KernelParameter$new(1, lower_bound = 0.1,
  upper_bound = 10, slice_sampling_scale = 4)

# A kernel parameter can be associated with any type of kernel
KernelPeriodic$new(period_length = const_param, lengthscale = samp_param)
```

---

 KernelPeriodic

*R6 class for Periodic Kernels*


---

**Description**

R6 class for Periodic Kernels

**Super class**

[BKTR::Kernel](#) -> KernelPeriodic

**Public fields**

lengthscale The lengthscale parameter instance of the kernel

period\_length The period length parameter instance of the kernel

has\_dist\_matrix Identify if the kernel has a distance matrix or not

name The kernel's name

**Methods****Public methods:**

- [KernelPeriodic\\$new\(\)](#)
- [KernelPeriodic\\$core\\_kernel\\_fn\(\)](#)
- [KernelPeriodic\\$clone\(\)](#)

**Method** `new()`: Create a new KernelPeriodic object.

*Usage:*

```
KernelPeriodic$new(
  lengthscale = KernelParameter$new(2),
  period_length = KernelParameter$new(2),
  kernel_variance = 1,
  jitter_value = NULL
)
```

*Arguments:*

`lengthscale` KernelParameter: The lengthscale parameter instance of the kernel  
`period_length` KernelParameter: The period length parameter instance of the kernel  
`kernel_variance` Numeric: The variance of the kernel  
`jitter_value` Numeric: The jitter value to add to the kernel matrix

*Returns:* A new KernelPeriodic object.

**Method** `core_kernel_fn()`: Method to compute the core kernel's covariance matrix

*Usage:*

```
KernelPeriodic$core_kernel_fn()
```

*Returns:* The core kernel's covariance matrix

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
KernelPeriodic$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

**Examples**

```
# Create a new Periodic kernel
k_periodic <- KernelPeriodic$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_periodic$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_periodic$kernel_gen()
```

KernelRQ

*R6 class for Rational Quadratic Kernels***Description**

R6 class for Rational Quadratic Kernels

**Super class**

[BKTR::Kernel](#) -> KernelRQ

**Public fields**

`lengthscale` The lengthscale parameter instance of the kernel

`alpha` The alpha parameter instance of the kernel

`has_dist_matrix` The distance matrix between points in a tensor format

`name` The kernel's name

**Methods****Public methods:**

- [KernelRQ\\$new\(\)](#)
- [KernelRQ\\$core\\_kernel\\_fn\(\)](#)
- [KernelRQ\\$clone\(\)](#)

**Method** `new()`: Create a new KernelRQ object.

*Usage:*

```
KernelRQ$new(
  lengthscale = KernelParameter$new(2),
  alpha = KernelParameter$new(2),
  kernel_variance = 1,
  jitter_value = NULL
)
```

*Arguments:*

`lengthscale` KernelParameter: The lengthscale parameter instance of the kernel

`alpha` KernelParameter: The alpha parameter instance of the kernel

`kernel_variance` Numeric: The variance of the kernel

`jitter_value` Numeric: The jitter value to add to the kernel matrix

*Returns:* A new KernelRQ object.

**Method** `core_kernel_fn()`: Method to compute the core kernel's covariance matrix

*Usage:*

```
KernelRQ$core_kernel_fn()
```

*Returns:* The core kernel's covariance matrix

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
KernelRQ$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## Examples

```
# Create a new RQ kernel
k_rq <- KernelRQ$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_rq$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_rq$kernel_gen()
```

---

KernelSE

*R6 class for Square Exponential Kernels*

---

## Description

R6 class for Square Exponential Kernels

## Super class

`BKTR::Kernel` -> KernelSE

## Public fields

`lengthscale` The lengthscale parameter instance of the kernel  
`has_dist_matrix` Identify if the kernel has a distance matrix or not  
`name` The kernel's name

## Methods

### Public methods:

- `KernelSE$new()`
- `KernelSE$core_kernel_fn()`
- `KernelSE$clone()`

**Method** `new()`: Create a new KernelSE object.

*Usage:*

```
KernelSE$new(
  lengthscale = KernelParameter$new(2),
  kernel_variance = 1,
  jitter_value = NULL
)
```

*Arguments:*

`lengthscale` `KernelParameter`: The lengthscale parameter instance of the kernel

`kernel_variance` `Numeric`: The variance of the kernel

`jitter_value` `Numeric`: The jitter value to add to the kernel matrix

*Returns:* A new `KernelSE` object.

**Method** `core_kernel_fn()`: Method to compute the core kernel's covariance matrix

*Usage:*

```
KernelSE$core_kernel_fn()
```

*Returns:* The core kernel's covariance matrix

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
KernelSE$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

**Examples**

```
# Create a new SE kernel
k_se <- KernelSE$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_se$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_se$kernel_gen()
```

---

KernelWhiteNoise

*R6 class for White Noise Kernels*


---

**Description**

R6 class for White Noise Kernels

**Super class**

[BKTR::Kernel](#) -> KernelWhiteNoise

**Public fields**

`has_dist_matrix` Identify if the kernel has a distance matrix or not  
`name` The kernel's name

**Methods****Public methods:**

- `KernelWhiteNoise$new()`
- `KernelWhiteNoise$core_kernel_fn()`
- `KernelWhiteNoise$clone()`

**Method new():**

*Usage:*

```
KernelWhiteNoise$new(kernel_variance = 1, jitter_value = NULL)
```

*Arguments:*

`kernel_variance` Numeric: The variance of the kernel

`jitter_value` Numeric: The jitter value to add to the kernel matrix

*Returns:* A new `KernelWhiteNoise` object.

**Method core\_kernel\_fn():** Method to compute the core kernel's covariance matrix

*Usage:*

```
KernelWhiteNoise$core_kernel_fn()
```

*Returns:* The core kernel's covariance matrix

**Method clone():** The objects of this class are cloneable with this method.

*Usage:*

```
KernelWhiteNoise$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

**Examples**

```
# Create a new white noise kernel
k_white_noise <- KernelWhiteNoise$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_white_noise$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_white_noise$kernel_gen()
```

---

plot\_beta\_dists      *Plot Beta Coefficients Distribution*

---

### Description

Plot the distribution of beta values for a given list of labels.

### Usage

```
plot_beta_dists(
  bktr_reg,
  labels_list,
  show_figure = TRUE,
  fig_width = 9,
  fig_height = 6,
  fig_resolution = 200
)
```

### Arguments

bktr_reg	BKTRRegressor: BKTRRegressor object.
labels_list	List: List of labels tuple (spatial, temporal, feature) for which to plot the beta distribution through iterations
show_figure	Boolean: Whether to show the figure. Defaults to True.
fig_width	Integer: Figure width. Defaults to 9.
fig_height	Integer: Figure height. Defaults to 6.
fig_resolution	Numeric: Figure resolution PPI. Defaults to 200.

### Value

ggplot or NULL: ggplot object or NULL if show\_figure is set to FALSE.

### Examples

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
bktr_regressor <- BKTRRegressor$new(
  data_df <- bixi_data$data_df,
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot temporal beta coefficients for the first station and the first feature
spa_lab <- bixi_data$spatial_positions_df$location[3]
plot_beta_dists(
  bktr_regressor,
```

```
labels_list = list(
  c(spa_lab, '2019-04-15', 'area_park'),
  c(spa_lab, '2019-04-16', 'area_park'),
  c(spa_lab, '2019-04-16', 'mean_temp_c')
),
)
```

---

plot\_covariates\_beta\_dists

*Plot Beta Coefficients Distribution Regrouped by Covariates*

---

## Description

Plot the distribution of beta estimates regrouped by covariates.

## Usage

```
plot_covariates_beta_dists(
  bktr_reg,
  feature_labels = NULL,
  show_figure = TRUE,
  fig_width = 9,
  fig_height = 6,
  fig_resolution = 200
)
```

## Arguments

**bktr\_reg** BKTRRegressor: BKTRRegressor object.

**feature\_labels** Array or NULL: Array of feature labels for which to plot the beta estimates distribution. If NULL plot for all features.

**show\_figure** Boolean: Whether to show the figure. Defaults to True.

**fig\_width** Integer: Figure width. Defaults to 9.

**fig\_height** Integer: Figure height. Defaults to 6.

**fig\_resolution** Numeric: Figure resolution PPI. Defaults to 200.

## Value

ggplot or NULL: ggplot object or NULL if show\_figure is set to FALSE.

**Examples**

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
bktr_regressor <- BKTRRegressor$new(
  formula = 'nb_departure ~ 1 + area_park + mean_temp_c + total_precip_mm',
  data_df <- bixi_data$data_df,
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot beta estimates distribution for all features
plot_covariates_beta_dists(bktr_regressor)
# Or plot for a subset of features
plot_covariates_beta_dists(bktr_regressor, c('area_park', 'mean_temp_c'))
```

---

plot\_hyperparams\_dists

*Plot Hyperparameters Distributions*

---

**Description**

Plot the distribution of hyperparameters through iterations

**Usage**

```
plot_hyperparams_dists(
  bktr_reg,
  hyperparameters = NULL,
  show_figure = TRUE,
  fig_width = 9,
  fig_height = 6,
  fig_resolution = 200
)
```

**Arguments**

bktr_reg	BKTRRegressor: BKTRRegressor object.
hyperparameters	Array or NULL: Array of hyperparameters to plot. If NULL, plot all hyperparameters. Defaults to NULL.
show_figure	Boolean: Whether to show the figure. Defaults to True.
fig_width	Integer: Figure width. Defaults to 9.
fig_height	Integer: Figure height. Defaults to 6.
fig_resolution	Numeric: Figure resolution PPI. Defaults to 200.

**Value**

ggplot or NULL: ggplot object or NULL if show\_figure is set to FALSE.

**Examples**

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
k_matern <- KernelMatern$new()
k_periodic <- KernelPeriodic$new()
bktr_regressor <- BKTRRegressor$new(
  data_df <- bixi_data$data_df,
  spatial_kernel = k_matern,
  temporal_kernel = k_periodic,
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot the distribution of all hyperparameters
plot_hyperparams_dists(bktr_regressor)

# Plot the distribution of the spatial kernel hyperparameters
spa_par_name <- paste0('Spatial - ', k_matern$parameters[[1]]$full_name)
plot_hyperparams_dists(bktr_regressor, spa_par_name)

# Plot the distribution of the temporal kernel hyperparameters
temp_par_names <- sapply(k_periodic$parameters, function(x) x$full_name)
temp_par_names <- paste0('Temporal - ', temp_par_names)
plot_hyperparams_dists(bktr_regressor, temp_par_names)
```

---

plot\_hyperparams\_traceplot

*Plot Hyperparameters Traceplot*

---

**Description**

Plot the evolution of hyperparameters through iterations. (Traceplot)

**Usage**

```
plot_hyperparams_traceplot(
  bktr_reg,
  hyperparameters = NULL,
  show_figure = TRUE,
  fig_width = 9,
  fig_height = 5.5,
  fig_resolution = 200
)
```

**Arguments**

**bktr\_reg**           BKTRRegressor: BKTRRegressor object.  
**hyperparameters**    Array or NULL: Array of hyperparameters to plot. If NULL, plot all hyperparameters. Defaults to NULL.  
**show\_figure**        Boolean: Whether to show the figure. Defaults to True.  
**fig\_width**           Integer: Figure width. Defaults to 9.  
**fig\_height**          Integer: Figure height. Defaults to 5.5.  
**fig\_resolution**    Numeric: Figure resolution PPI. Defaults to 200.

**Value**

ggplot or NULL: ggplot object or NULL if show\_figure is set to FALSE.

**Examples**

```

# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
k_matern <- KernelMatern$new()
k_periodic <- KernelPeriodic$new()
bktr_regressor <- BKTRRegressor$new(
  data_df <- bixi_data$data_df,
  spatial_kernel = k_matern,
  temporal_kernel = k_periodic,
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot the traceplot of all hyperparameters
plot_hyperparams_traceplot(bktr_regressor)

# Plot the traceplot of the spatial kernel hyperparameters
spa_par_name <- paste0('Spatial - ', k_matern$parameters[[1]]$full_name)
plot_hyperparams_traceplot(bktr_regressor, spa_par_name)

# Plot the traceplot of the temporal kernel hyperparameters
temp_par_names <- sapply(k_periodic$parameters, function(x) x$full_name)
temp_par_names <- paste0('Temporal - ', temp_par_names)
plot_hyperparams_traceplot(bktr_regressor, temp_par_names)

```

**Description**

Create a plot of beta values through space for a given temporal point and a set of feature labels. We use ggmap under the hood, so you need to provide a Google or Stadia API token to plot on a map. See: <https://cran.r-project.org/web/packages/ggmap/readme/README.html> for more details on how to get an API token.

**Usage**

```
plot_spatial_betas(
  bktr_reg,
  plot_feature_labels,
  temporal_point_label,
  nb_cols = 1,
  use_dark_mode = TRUE,
  show_figure = TRUE,
  zoom = 11,
  google_token = NULL,
  stadia_token = NULL,
  fig_width = 8.5,
  fig_height = 5.5,
  fig_resolution = 200
)
```

**Arguments**

bktr_reg	BKTRRegressor: BKTRRegressor object.
plot_feature_labels	Array: Array of feature labels to plot.
temporal_point_label	String: Temporal point label to plot.
nb_cols	Integer: The number of columns to use in the facet grid.
use_dark_mode	Boolean: Whether to use a dark mode for the geographic map or not. Defaults to TRUE.
show_figure	Boolean: Whether to show the figure. Defaults to True.
zoom	Integer: Zoom level for the geographic map. Defaults to 11.
google_token	String or NULL: Google API token to use for the geographic map. Defaults to NULL.
stadia_token	String or NULL: Stadia API token to use for the geographic map. Defaults to NULL.
fig_width	Numeric: Figure width when figure is shown. Defaults to 8.5.
fig_height	Numeric: Figure height when figure is shown. Defaults to 5.5.
fig_resolution	Numeric: Figure resolution PPI. Defaults to 200.

**Value**

ggplot or NULL: ggplot object or NULL if show\_figure is set to FALSE.

**Examples**

```

# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
bktr_regressor <- BKTRRegressor$new(
  data_df <- bixi_data$data_df,
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot spatial beta coefficients for the first time point and the two features
# Use your own Google API token instead of 'GOOGLE_API_TOKEN'
plot_spatial_betas(
  bktr_regressor,
  plot_feature_labels = c('mean_temp_c', 'area_park'),
  temporal_point_label = bixi_data$temporal_positions_df$time[1],
  google_token = 'GOOGLE_API_TOKEN')

# We can also use light mode and plot the maps side by side
# Use your own Stadia API token instead of 'STADIA_API_TOKEN'
plot_spatial_betas(
  bktr_regressor,
  plot_feature_labels = c('mean_temp_c', 'area_park', 'total_precip_mm'),
  temporal_point_label = bixi_data$temporal_positions_df$time[10],
  use_dark_mode = FALSE, nb_cols = 3, stadia_token = 'STADIA_API_TOKEN')

```

---

plot\_temporal\_betas    *Plot Temporal Beta Coefficients*

---

**Description**

Create a plot of the beta values through time for a given spatial point and a set of feature labels.

**Usage**

```

plot_temporal_betas(
  bktr_reg,
  plot_feature_labels,
  spatial_point_label,
  date_format = "%Y-%m-%d",
  show_figure = TRUE,
  fig_width = 8.5,
  fig_height = 5.5,
  fig_resolution = 200
)

```

**Arguments**

**bktr\_reg** BKTRRegressor: BKTRRegressor object.  
**plot\_feature\_labels** Array: Array of feature labels to plot.  
**spatial\_point\_label** String: Spatial point label to plot.  
**date\_format** String: Format of the date to use in bktr dataframes for the time. Defaults to '%Y-%m-%d'.  
**show\_figure** Boolean: Whether to show the figure. Defaults to True.  
**fig\_width** Numeric: Figure width when figure is shown. Defaults to 8.5.  
**fig\_height** Numeric: Figure height when figure is shown. Defaults to 5.5.  
**fig\_resolution** Numeric: Figure resolution PPI. Defaults to 200.

**Value**

ggplot or NULL: ggplot object or NULL if show\_figure is set to FALSE.

**Examples**

```

# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
bktr_regressor <- BKTRRegressor$new(
  data_df <- bixi_data$data_df,
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot temporal beta coefficients for the first station and the two features
plot_temporal_betas(
  bktr_regressor,
  plot_feature_labels = c('mean_temp_c', 'area_park'),
  spatial_point_label = bixi_data$spatial_positions_df$location[1])

```

---

plot\_y\_estimates      *Plot Y Estimates*

---

**Description**

Plot y estimates vs observed y values.

**Usage**

```
plot_y_estimates(
  bktr_reg,
  show_figure = TRUE,
  fig_width = 5,
  fig_height = 5,
  fig_resolution = 200,
  fig_title = "y estimates vs observed y values"
)
```

**Arguments**

bktr_reg	BKTRRegressor: BKTRRegressor object.
show_figure	Boolean: Whether to show the figure. Defaults to True.
fig_width	Numeric: Figure width when figure is shown. Defaults to 5.
fig_height	Numeric: Figure height when figure is shown. Defaults to 5.
fig_resolution	Numeric: Figure resolution PPI when figure is shown. Defaults to 200.
fig_title	String or NULL: Figure title if provided. Defaults to 'y estimates vs observed y values'

**Value**

ggplot or NULL: ggplot object or NULL if show\_figure is set to FALSE.

**Examples**

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
bktr_regressor <- BKTRRegressor$new(
  data_df <- bixi_data$data_df,
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot Y estimates vs observed y values
plot_y_estimates(bktr_regressor)
```

---

print.BKTRRegressor    *Print the summary of a BKTRRegressor instance*

---

**Description**

Print the summary of a BKTRRegressor instance

**Usage**

```
## S3 method for class 'BKTRRegressor'
print(x, ...)
```

**Arguments**

x                    A BKTRRegressor instance  
 ...                  Additional arguments to comply with generic function

---

reshape\_covariate\_dfs *Function used to transform covariates coming from two dataframes one for spatial and one for temporal into a single dataframe with the right shape for the BKTR Regressor. This is useful when the temporal covariates do not vary trough space and the spatial covariates do not vary trough time (Like in the BIXI example). The function also adds a column for the target variable at the beginning of the dataframe.*

---

**Description**

Function used to transform covariates coming from two dataframes one for spatial and one for temporal into a single dataframe with the right shape for the BKTR Regressor. This is useful when the temporal covariates do not vary trough space and the spatial covariates do not vary trough time (Like in the BIXI example). The function also adds a column for the target variable at the beginning of the dataframe.

**Usage**

```
reshape_covariate_dfs(spatial_df, temporal_df, y_df, y_column_name = "y")
```

**Arguments**

spatial\_df        data.table: Spatial covariates dataframe with an index named location and a shape of (n\_locations, n\_spatial\_covariates)  
 temporal\_df      data.table: Temporal covariates dataframe with an index named time and a shape of (n\_times, n\_temporal\_covariates)  
 y\_df             data.table: The dataframe containing the target variable. It should have a shape of (n\_locations, n\_times). The columns and index names of this dataframe should be correspond to the one of the spatial\_df and temporal\_df.  
 y\_column\_name    string: The name of the target variable column in y\_df. Default to 'y'.

**Value**

data.table: The reshaped covariates dataframe with a shape of (n\_locations \* n\_times, 1 + n\_spatial\_covariates + n\_temporal\_covariates). The first two columns are the indexes (location, time), the following column is the target variable and the other columns are the covariates.

**Examples**

```
# Let's reshape the BIXI dataframes without normalization
new_data_df <- reshape_covariate_dfs(
  spatial_df = BKTR::bixi_spatial_features,
  temporal_df = BKTR::bixi_temporal_features,
  y_df = BKTR::bixi_station_departures,
  y_column_name = 'whole_nb_departure')
# The resulting dataframe has the right shape to be a BKTRRegressor data_df
head(new_data_df)
```

---

```
simulate_spatiotemporal_data
```

*Simulate Spatiotemporal Data Using Kernel Covariances.*

---

**Description**

Simulate Spatiotemporal Data Using Kernel Covariances.

**Usage**

```
simulate_spatiotemporal_data(
  nb_locations,
  nb_time_points,
  nb_spatial_dimensions,
  spatial_scale,
  time_scale,
  spatial_covariates_means,
  temporal_covariates_means,
  spatial_kernel,
  temporal_kernel,
  noise_variance_scale
)
```

**Arguments**

nb\_locations Integer: Number of spatial locations  
 nb\_time\_points Integer: Number of time points  
 nb\_spatial\_dimensions Integer: Number of spatial dimensions  
 spatial\_scale Numeric: Spatial scale  
 time\_scale Numeric: Time scale  
 spatial\_covariates\_means Vector: Spatial covariates means  
 temporal\_covariates\_means Vector: Temporal covariates means

```

spatial_kernel Kernel: Spatial kernel
temporal_kernel
                Kernel: Temporal kernel
noise_variance_scale
                Numeric: Noise variance scale

```

**Value**

A list containing 4 dataframes: - 'data\_df' contains the response variable and the covariates - 'spatial\_positions\_df' contains the spatial locations and their coordinates - 'temporal\_positions\_df' contains the time points and their coordinates - 'beta\_df' contains the true beta coefficients

**Examples**

```

# Simulate data with 20 locations, 30 time points, in 2D spatial dimensions
# with 3 spatial covariates and 2 temporal covariates
simu_data <- simulate_spatiotemporal_data(
  nb_locations=20,
  nb_time_points=30,
  nb_spatial_dimensions=2,
  spatial_scale=10,
  time_scale=10,
  spatial_covariates_means=c(0, 2, 4),
  temporal_covariates_means=c(1, 3),
  spatial_kernel=KernelMatern$new(),
  temporal_kernel=KernelSE$new(),
  noise_variance_scale=1)

# The dataframes are similar to bixi_data, we have:
# - data_df
head(simu_data$data_df)
# - spatial_positions_df
head(simu_data$spatial_positions_df)
# - temporal_positions_df
head(simu_data$temporal_positions_df)

# We also obtain the true beta coefficients used to simulate the data
head(simu_data$beta_df)

```

---

summary.BKTRRegressor *Summarize a BKTRRegressor instance*

---

**Description**

Summarize a BKTRRegressor instance

**Usage**

```
## S3 method for class 'BKTRRegressor'
summary(object, ...)
```

**Arguments**

object	A BKTRRegressor instance
...	Additional arguments to comply with generic function

---

TensorOperator	<i>R6 singleton that contains the configuration for the tensor backend</i>
----------------	--

---

**Description**

Tensor backend configuration and methods for all the tensor operations in BKTR

**Super class**

[R6P::Singleton](#) -> TensorOperator

**Public fields**

fp_type	The floating point type to use for the tensor operations
fp_device	The device to use for the tensor operations

**Methods****Public methods:**

- [TensorOperator\\$new\(\)](#)
- [TensorOperator\\$set\\_params\(\)](#)
- [TensorOperator\\$get\\_default\\_jitter\(\)](#)
- [TensorOperator\\$tensor\(\)](#)
- [TensorOperator\\$is\\_tensor\(\)](#)
- [TensorOperator\\$eye\(\)](#)
- [TensorOperator\\$ones\(\)](#)
- [TensorOperator\\$zeros\(\)](#)
- [TensorOperator\\$rand\(\)](#)
- [TensorOperator\\$randn\(\)](#)
- [TensorOperator\\$randn\\_like\(\)](#)
- [TensorOperator\\$arange\(\)](#)
- [TensorOperator\\$rand\\_choice\(\)](#)
- [TensorOperator\\$kronecker\\_prod\(\)](#)
- [TensorOperator\\$khatri\\_rao\\_prod\(\)](#)
- [TensorOperator\\$clone\(\)](#)

**Method** `new()`: Initialize the tensor operator with the given floating point type and device

*Usage:*

```
TensorOperator$new(fp_type = "float64", fp_device = "cpu")
```

*Arguments:*

`fp_type` The floating point type to use for the tensor operations (either "float64" or "float32")

`fp_device` The device to use for the tensor operations (either "cpu" or "cuda")

*Returns:* A new tensor operator instance

**Method** `set_params()`: Set the tensor operator parameters

*Usage:*

```
TensorOperator$set_params(fp_type = NULL, fp_device = NULL, seed = NULL)
```

*Arguments:*

`fp_type` The floating point type to use for the tensor operations (either "float64" or "float32")

`fp_device` The device to use for the tensor operations (either "cpu" or "cuda")

`seed` The seed to use for the random number generator

**Method** `get_default_jitter()`: Get the default jitter value for the floating point type used by the tensor operator

*Usage:*

```
TensorOperator$get_default_jitter()
```

*Returns:* The default jitter value for the floating point type used by the tensor operator

**Method** `tensor()`: Create a tensor from a vector or matrix of data with the tensor operator dtype and device

*Usage:*

```
TensorOperator$tensor(tensor_data)
```

*Arguments:*

`tensor_data` The vector or matrix of data to create the tensor from

*Returns:* A new tensor with the tensor operator dtype and device

**Method** `is_tensor()`: Check if a provided object is a tensor

*Usage:*

```
TensorOperator$is_tensor(tensor)
```

*Arguments:*

`tensor` The object to check

*Returns:* A boolean indicating if the object is a tensor

**Method** `eye()`: Create a tensor with a diagonal of ones and zeros with the tensor operator dtype and device for a given dimension

*Usage:*

```
TensorOperator$eye(eye_dim)
```

*Arguments:*

`eye_dim` The dimension of the tensor to create

*Returns:* A new tensor with a diagonal of ones and zeros with the tensor operator dtype and device

**Method** `ones()`: Create a tensor of ones with the tensor operator dtype and device for a given dimension

*Usage:*

```
TensorOperator$ones(tsr_dim)
```

*Arguments:*

`tsr_dim` The dimension of the tensor to create

*Returns:* A new tensor of ones with the tensor operator dtype and device

**Method** `zeros()`: Create a tensor of zeros with the tensor operator dtype and device for a given dimension

*Usage:*

```
TensorOperator$zeros(tsr_dim)
```

*Arguments:*

`tsr_dim` The dimension of the tensor to create

*Returns:* A new tensor of zeros with the tensor operator dtype and device

**Method** `rand()`: Create a tensor of random uniform values with the tensor operator dtype and device for a given dimension

*Usage:*

```
TensorOperator$rand(tsr_dim)
```

*Arguments:*

`tsr_dim` The dimension of the tensor to create

*Returns:* A new tensor of random values with the tensor operator dtype and device

**Method** `randn()`: Create a tensor of random normal values with the tensor operator dtype and device for a given dimension

*Usage:*

```
TensorOperator$randn(tsr_dim)
```

*Arguments:*

`tsr_dim` The dimension of the tensor to create

*Returns:* A new tensor of random normal values with the tensor operator dtype and device

**Method** `randn_like()`: Create a tensor of random uniform values with the same shape as a given tensor with the tensor operator dtype and device

*Usage:*

```
TensorOperator$randn_like(input_tensor)
```

*Arguments:*

`input_tensor` The tensor to use as a shape reference

*Returns:* A new tensor of random uniform values with the same shape as a given tensor

**Method** `arange()`: Create a tensor of a range of values with the tensor operator dtype and device for a given start and end

*Usage:*

```
TensorOperator$arange(start, end)
```

*Arguments:*

`start` The start of the range  
`end` The end of the range

*Returns:* A new tensor of a range of values with the tensor operator dtype and device

**Method** `rand_choice()`: Choose random values from a tensor for a given number of samples

*Usage:*

```
TensorOperator$rand_choice(  
  choices_tsr,  
  nb_sample,  
  use_replace = FALSE,  
  weights_tsr = NULL  
)
```

*Arguments:*

`choices_tsr` The tensor to choose values from  
`nb_sample` The number of samples to choose  
`use_replace` A boolean indicating if the sampling should be done with replacement. Defaults to FALSE  
`weights_tsr` The weights to use for the sampling. If NULL, the sampling is uniform. Defaults to NULL

*Returns:* A new tensor of randomly chosen values from a tensor

**Method** `kronecker_prod()`: Efficiently compute the kronecker product of two matrices in tensor format

*Usage:*

```
TensorOperator$kronecker_prod(a, b)
```

*Arguments:*

`a` The first tensor  
`b` The second tensor

*Returns:* The kronecker product of the two matrices

**Method** `khatri_rao_prod()`: Efficiently compute the khatri rao product of two matrices in tensor format having the same number of columns

*Usage:*

```
TensorOperator$khatri_rao_prod(a, b)
```

*Arguments:*

`a` The first tensor

b The second tensor

*Returns:* The khatri rao product of the two matrices

**Method** `clone()`: The objects of this class are cloneable with this method.

*Usage:*

```
TensorOperator$clone(deep = FALSE)
```

*Arguments:*

`deep` Whether to make a deep clone.

## Examples

```
# Set the seed, setup the tensor floating point type and device
TSR$set_params(fp_type='float64', fp_device='cpu', seed=42)
# Create a tensor from a vector
TSR$tensor(c(1, 2, 3))
# Create a tensor from a matrix
TSR$tensor(matrix(c(1, 2, 3, 4), nrow=2))
# Create a 3x3 tensor with a diagonal of ones and zeros elsewhere
TSR$eye(3)
# Create a tensor of ones (with 6 elements, 2 rows and 3 columns)
TSR$ones(c(2, 3))
# Create a tensor of zeros (with 12 elements, 3 rows and 4 columns)
TSR$zeros(c(3, 4))
# Create a tensor of random uniform values (with 6 elements)
TSR$rand(c(2, 3))
# Create a tensor of random normal values (with 6 elements)
TSR$randn(c(2, 3))
# Create a tensor of random normal values with the same shape as a given tensor
tsr_a <- TSR$randn(c(2, 3))
TSR$randn_like(tsr_a)
# Create a tensor of a range of values (1, 2, 3, 4)
TSR$range(1, 4)
# Choose two random values from a given tensor without replacement
tsr_b <- TSR$rand(6)
TSR$rand_choice(tsr_b, 2)
# Use the tensor operator to compute the kronecker product of two 2x2 matrices
tsr_c <- TSR$tensor(matrix(c(1, 2, 3, 4), nrow=2))
tsr_d <- TSR$tensor(matrix(c(5, 6, 7, 8), nrow=2))
TSR$kronecker_prod(tsr_c, tsr_d) # Returns a 4x4 tensor
# Use the tensor operator to compute the khatri rao product of two 2x2 matrices
TSR$khatri_rao_prod(tsr_c, tsr_d) # Returns a 4x2 tensor
# Check if a given object is a tensor
TSR$is_tensor(tsr_d) # Returns TRUE
TSR$is_tensor(TSR$eye(2)) # Returns TRUE
TSR$is_tensor(1) # Returns FALSE
```

---

TSR

*Tensor Operator Singleton*

---

**Description**

Singleton instance of the TensorOperator class that contains all informations related the tensor API; tensor methods, used data type and used device.

**Usage**

TSR

**Format**

An object of class TensorOperator (inherits from Singleton, R6) of length 19.

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