

index.DB(clusterSim)

Davies-Bouldin's index

$$DB(u) = \frac{1}{u} \sum_{r=1}^u \max_{\substack{s \\ r \neq s}} \left(\frac{S_r + S_s}{d_{rs}} \right),$$

where: $r, s = 1, \dots, u$ – cluster number,

u – number of clusters ($u \geq 2$),

$i, k = 1, \dots, n$ – object number,

n – number of objects,

P_r, P_s – r -th, s -th cluster,

$\mathbf{z}_r = (z_{r1}, \dots, z_{rm})$ – centroid or medoid of cluster P_r ,

$j = 1, \dots, m$ – variable number,

$d_{rs} = \sqrt[p]{\sum_{j=1}^m |z_{rj} - z_{sj}|^p}$ – distance between centroids or medoids of clusters P_r and P_s (for $p = 1$ Manhattan distance, for $p = 2$ Euclidean distance),

$S_r = \sqrt[q]{\frac{1}{n_r} \sum_{i \in P_r} \sum_{j=1}^m |x_{ij}^r - z_{rj}|^q}$ – dispersion measure of a cluster P_r (for $q = 1$ the average distance of objects in cluster P_r to the centroid or medoid of cluster P_r ; for $q = 2$ the standard deviation of the distance of objects in cluster P_r to the centroid or medoid of cluster P_r),
 n_r (n_s) – number of objects in cluster P_r (P_s).

The value of u , which minimizes $DB(u)$, is regarded as specifying the number of clusters.

The Davies-Bouldin's index have two limitations:

- index is not permitted if the number of clusters equals the number of objects ($DB(u) = 0$ if $u = n$),
- the certain number of clusters to have only one object is allowed. An unlimited number of single member clusters is not permitted.

$DB(u)$ is a very good cluster separation measure if each cluster contain at least two objects.

References

Davies, D.L., Bouldin, D.W. (1979), *A cluster separation measure*, “IEEE Transactions on Pattern Analysis and Machine Intelligence”, vol. 1, no. 2, 224-227.