

# Knee Injuries - Marginal Models

February 1, 2012

First the dataset knee is loaded:

```
> library(catdata)
> data(knee)
> attach(knee)
```

To obtain a simple binary model the response variables are dichotomized. The groups are constructed by pain level up to level 2 und pain level higher than level 2.

```
> R2D <- rep(0, length(R2))
> R3D <- rep(0, length(R3))
> R4D <- rep(0, length(R3))
> R2D[R2>2] <- 1
> R3D[R3>2] <- 1
> R4D[R4>2] <- 1
```

Now the covariates have to be transformed so that they can be used for the functions "gee" from the "gee"-library and "geeglm" from the "geepack"-library, which will be employed for fitting the models.

```
> N <- rep(knee$N, each=3)
> Th <- rep(knee$Th, each=3)
> Age <- rep(knee$Age, each=3)
> Sex <- rep(knee$Sex, each=3)
```

Now the response vector is built and the quadratic age-effect "Age2" is computed.

```
> Response <- c(rbind(R2D,R3D,R4D))
> Age2 <- Age^2
```

The covariates therapy and sex are treated as factors:

```
> Th <- as.factor(Th)
> Sex <- as.factor(Sex)
```

First the GEEs are fitted with the funtion "gee" from library "gee".

```
> library(gee)
```

The first model is a GEE with independent correlation structure:

```

> gee1a <- gee(Response ~ Th + Sex + Age + Age2, id=N,
+ family=binomial(link=logit))

> summary(gee1a)

GEE:  GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)

Model:
Link:                      Logit
Variance to Mean Relation: Binomial
Correlation Structure:     Independent

Call:
gee(formula = Response ~ Th + Sex + Age + Age2, id = N, family = binomial(link = logit))

Summary of Residuals:
      Min       1Q   Median       3Q      Max
-0.8093879 -0.4771034  0.2393885  0.3881349  0.7660757

Coefficients:
              Estimate Naive S.E.   Naive z Robust S.E.   Robust z
(Intercept) -4.749831307 1.279009755 -3.713679 1.821208734 -2.6080653
Th2          -0.673979165 0.223825316 -3.011184 0.334205312 -2.0166620
Sex1         0.265689239 0.241933592  1.098191 0.366733338  0.7244753
Age          0.381150842 0.087570504  4.352503 0.125833185  3.0290169
Age2        -0.006124345 0.001381627 -4.432705 0.001984839 -3.0855622

Estimated Scale Parameter: 1.019152
Number of Iterations: 1

Working Correlation
      [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1

```

The second model is a GEE with exchangeable correlation structure:

```

> gee2a <- gee(Response ~ Th + Sex + Age + Age2, id=N,
+ family=binomial(link=logit), corstr="exchangeable")

> summary(gee2a)

GEE:  GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)

Model:
Link:                      Logit
Variance to Mean Relation: Binomial

```

Correlation Structure:      Exchangeable

Call:

```
gee(formula = Response ~ Th + Sex + Age + Age2, id = N, family = binomial(link = logit),
    corstr = "exchangeable")
```

Summary of Residuals:

	Min	1Q	Median	3Q	Max
	-0.8093879	-0.4771034	0.2393885	0.3881349	0.7660757

Coefficients:

	Estimate	Naive S.E.	Naive z	Robust S.E.	Robust z
(Intercept)	-4.749831307	1.903793545	-2.4949298	1.821208734	-2.6080653
Th2	-0.673979165	0.333161800	-2.0229785	0.334205312	-2.0166620
Sex1	0.265689239	0.360115792	0.7377884	0.366733338	0.7244753
Age	0.381150842	0.130347841	2.9241055	0.125833185	3.0290169
Age2	-0.006124345	0.002056538	-2.9779873	0.001984839	-3.0855622

Estimated Scale Parameter: 1.019152

Number of Iterations: 1

Working Correlation

	[,1]	[,2]	[,3]
[1,]	1.0000000	0.6078016	0.6078016
[2,]	0.6078016	1.0000000	0.6078016
[3,]	0.6078016	0.6078016	1.0000000

Finally a GEE with exponential correlation structure is fitted:

```
> gee3a <- gee(Response ~ Th + Sex + Age + Age2, id=N,
+ family=binomial(link=logit), corstr="AR-M", Mv=1)
```

```
> summary(gee3a)
```

GEE: GENERALIZED LINEAR MODELS FOR DEPENDENT DATA  
gee S-function, version 4.13 modified 98/01/27 (1998)

Model:

Link:                              Logit  
Variance to Mean Relation: Binomial  
Correlation Structure:      AR-M , M = 1

Call:

```
gee(formula = Response ~ Th + Sex + Age + Age2, id = N, family = binomial(link = logit),
    corstr = "AR-M", Mv = 1)
```

Summary of Residuals:

	Min	1Q	Median	3Q	Max
	-0.8061636	-0.4668263	0.2354196	0.3833613	0.7933803

Coefficients:

	Estimate	Naive S.E.	Naive z	Robust S.E.	Robust z
(Intercept)	-4.72614143	1.912768743	-2.4708379	1.784861526	-2.6479037
Th2	-0.74849866	0.333910055	-2.2416176	0.328084283	-2.2814219
Sex1	0.19277195	0.361954995	0.5325854	0.362465544	0.5318352
Age	0.38489413	0.131285068	2.9317434	0.123289666	3.1218685
Age2	-0.00621548	0.002074949	-2.9954850	0.001945528	-3.1947525

Estimated Scale Parameter: 1.018095

Number of Iterations: 3

Working Correlation

	[,1]	[,2]	[,3]
[1,]	1.0000000	0.7058422	0.4982131
[2,]	0.7058422	1.0000000	0.7058422
[3,]	0.4982131	0.7058422	1.0000000

In the following the corresponding marginal models are fitted with the function "geeglm" from the library "geepack".

```
> library(geepack)
```

Model with independent correlation structure:

```
> gee1b <- geeglm(Response ~ Th + Sex + Age + Age2, id=N,
+ family=binomial(link=logit))
```

```
> summary(gee1b)
```

Call:

```
geeglm(formula = Response ~ Th + Sex + Age + Age2, family = binomial(link = logit),
      id = N)
```

Coefficients:

	Estimate	Std.err	Wald	Pr(> W )
(Intercept)	-4.749831	1.821209	6.802	0.00911 **
Th2	-0.673979	0.334205	4.067	0.04373 *
Sex1	0.265689	0.366733	0.525	0.46877
Age	0.381151	0.125833	9.175	0.00245 **
Age2	-0.006124	0.001985	9.521	0.00203 **

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Estimated Scale Parameters:

	Estimate	Std.err
(Intercept)	1.006	0.103

Correlation: Structure = independenceNumber of clusters: 127 Maximum cluster size: 3

Model with exchangeable correlation structure:

```
> gee2b <- geeglm(Response ~ Th + Sex + Age + Age2, id=N,
+ family=binomial(link=logit), corstr="exchangeable")
```

```
> summary(gee2b)
```

Call:

```
geeglm(formula = Response ~ Th + Sex + Age + Age2, family = binomial(link = logit),
      id = N, corstr = "exchangeable")
```

Coefficients:

	Estimate	Std.err	Wald	Pr(> W )	
(Intercept)	-4.74983	1.82121	6.80	0.0091	**
Th2	-0.67398	0.33421	4.07	0.0437	*
Sex1	0.26569	0.36673	0.52	0.4688	
Age	0.38115	0.12583	9.17	0.0025	**
Age2	-0.00612	0.00198	9.52	0.0020	**

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Estimated Scale Parameters:

	Estimate	Std.err
(Intercept)	1.01	0.103

Correlation: Structure = exchangeable Link = identity

Estimated Correlation Parameters:

	Estimate	Std.err
alpha	0.608	0.0883

Number of clusters: 127 Maximum cluster size: 3

Model with exponential correlation structure:

```
> gee3b <- geeglm(Response ~ Th + Sex + Age + Age2, id=N,
+ family=binomial(link=logit), corstr="ar1")
```

```
> summary(gee3b)
```

Call:

```
geeglm(formula = Response ~ Th + Sex + Age + Age2, family = binomial(link = logit),
      id = N, corstr = "ar1")
```

Coefficients:

	Estimate	Std.err	Wald	Pr(> W )	
(Intercept)	-4.72712	1.78605	7.00	0.0081	**
Th2	-0.74443	0.32828	5.14	0.0233	*
Sex1	0.19674	0.36257	0.29	0.5874	
Age	0.38467	0.12338	9.72	0.0018	**
Age2	-0.00621	0.00195	10.17	0.0014	**

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

```
Estimated Scale Parameters:
      Estimate Std.err
(Intercept)      1  0.102
```

```
Correlation: Structure = ar1 Link = identity
```

```
Estimated Correlation Parameters:
      Estimate Std.err
alpha    0.676  0.0766
Number of clusters: 127 Maximum cluster size: 3
```

For comparison a simple GLM with logit-link is fitted with the same covariates as in the marginal models above:

```
> glm1 <- glm(Response ~ Th + Sex + Age + Age2,
+ family=binomial(link=logit))
> summary(glm1)
```

Call:

```
glm(formula = Response ~ Th + Sex + Age + Age2, family = binomial(link = logit))
```

```
Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.821  -1.139   0.740   0.991   1.705
```

Coefficients:

```
      Estimate Std. Error z value Pr(>|z|)
(Intercept) -4.74983    1.26693  -3.75  0.00018 ***
Th2          -0.67398    0.22171  -3.04  0.00237 **
Sex1         0.26569    0.23965   1.11  0.26758
Age          0.38115    0.08674   4.39  1.1e-05 ***
Age2        -0.00612    0.00137  -4.47  7.6e-06 ***
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(Dispersion parameter for binomial family taken to be 1)

```
Null deviance: 513.32 on 380 degrees of freedom
Residual deviance: 478.61 on 376 degrees of freedom
AIC: 488.6
```

Number of Fisher Scoring iterations: 4

It is often advantageous to center the variables like age around a value in the middle of its range. So now the marginal models from above are replicated with age centered around 30 years.

```
> Age <- Age-30
> Age2 <- Age^2
```

Again we use the function "gee" from the "gee"-library for fitting those models.

Model with independent correlation structure and centered age:

```
> gee1c <- gee(Response ~ Th + Sex + Age + Age2, id=N,
+ family=binomial(link=logit))
```

```
> summary(gee1c)
```

```
GEE:  GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)
```

Model:

```
Link:                               Logit
Variance to Mean Relation: Binomial
Correlation Structure:      Independent
```

Call:

```
gee(formula = Response ~ Th + Sex + Age + Age2, id = N, family = binomial(link = logit))
```

Summary of Residuals:

	Min	1Q	Median	3Q	Max
	-0.809	-0.477	0.239	0.388	0.766

Coefficients:

	Estimate	Naive S.E.	Naive z	Robust S.E.	Robust z
(Intercept)	1.17278	0.28476	4.12	0.44806	2.617
Th2	-0.67398	0.22383	-3.01	0.33421	-2.017
Sex1	0.26569	0.24193	1.10	0.36673	0.724
Age	0.01369	0.01204	1.14	0.01736	0.789
Age2	-0.00612	0.00138	-4.43	0.00198	-3.086

Estimated Scale Parameter: 1.02

Number of Iterations: 1

Working Correlation

	[,1]	[,2]	[,3]
[1,]	1	0	0
[2,]	0	1	0
[3,]	0	0	1

Model with exchangeable correlation structure and centered age:

```
> gee2c <- gee(Response ~ Th + Sex + Age + Age2, id=N,
+ family=binomial(link=logit), corstr="exchangeable")
```

```
> summary(gee2c)
```

```
GEE:  GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)
```

```

Model:
  Link:                               Logit
  Variance to Mean Relation: Binomial
  Correlation Structure:      Exchangeable

Call:
gee(formula = Response ~ Th + Sex + Age + Age2, id = N, family = binomial(link = logit),
    corstr = "exchangeable")

Summary of Residuals:
      Min       1Q   Median       3Q      Max
-0.809 -0.477  0.239  0.388  0.766

Coefficients:
              Estimate Naive S.E. Naive z Robust S.E. Robust z
(Intercept)  1.17278      0.42386   2.767      0.44806   2.617
Th2          -0.67398      0.33316  -2.023      0.33421  -2.017
Sex1          0.26569      0.36012   0.738      0.36673   0.724
Age           0.01369      0.01792   0.764      0.01736   0.789
Age2         -0.00612      0.00206  -2.978      0.00198  -3.086

Estimated Scale Parameter:  1.02
Number of Iterations:  1

Working Correlation
      [,1] [,2] [,3]
[1,] 1.000 0.608 0.608
[2,] 0.608 1.000 0.608
[3,] 0.608 0.608 1.000

Model with exponential correlation structure and centered age:

> gee3c <- gee(Response ~ Th + Sex + Age + Age2, id=N,
+ family=binomial(link=logit), corstr="AR-M", Mv=1)

> summary(gee3c)

GEE:  GENERALIZED LINEAR MODELS FOR DEPENDENT DATA
gee S-function, version 4.13 modified 98/01/27 (1998)

Model:
  Link:                               Logit
  Variance to Mean Relation: Binomial
  Correlation Structure:      AR-M , M = 1

Call:
gee(formula = Response ~ Th + Sex + Age + Age2, id = N, family = binomial(link = logit),
    corstr = "AR-M", Mv = 1)

```



Summary of Residuals:

Min	1Q	Median	3Q	Max
-0.806	-0.467	0.235	0.383	0.793

Coefficients:

	Estimate	Naive S.E.	Naive z	Robust S.E.	Robust z
(Intercept)	1.22675	0.42743	2.870	0.44356	2.766
Th2	-0.74850	0.33391	-2.242	0.32808	-2.281
Sex1	0.19277	0.36195	0.533	0.36247	0.532
Age	0.01197	0.01797	0.666	0.01699	0.704
Age2	-0.00622	0.00207	-2.995	0.00195	-3.195

Estimated Scale Parameter: 1.02

Number of Iterations: 3

Working Correlation

	[,1]	[,2]	[,3]
[1,]	1.000	0.706	0.498
[2,]	0.706	1.000	0.706
[3,]	0.498	0.706	1.000